1 INTRODUCTION

At a time when cars could only travel at a brisk walking speed and trucks had yet to be invented there was no need for superelevation. Even the tightest radius could be followed with comfort on a road that had only been sufficiently cambered to allow for drainage. As was inevitable, vehicle speeds went up and it was found that drivers were moving to the wrong side of the road on curves to take advantage of the camber and thus negotiate the curve at a greater speed in comfort. Brooklands was a famous racing circuit between the two World Wars, and one of its claims to fame was the heavy banking provided on the curves. It did not take all that long for designers to realise that something similar could, with advantage, be provided on public roads and superelevation was born. Two advantages were foreseen, being that

- ❑ tighter radii could be employed without adversely affecting the selection of design speed
- ❑ drivers could perhaps be persuaded to stay on their side of the road

With regard to the latter, it should be borne in mind that, at this time, South Africa made one of its greatest contributions to highway engineering because it was here that the painted centre line was invented. Drivers thus no longer had any excuse for not knowing where, relative to the road surface, they should be and the intention of the designers was that, for preference, this should include above it.

As far as the driver is concerned, there is actually no need for superelevation. All he wants to do is to effect a change of direction without having to change his selected speed and in comfort. A sufficiently large radius can achieve this state of affairs and has the added benefit that the route between two points is shortened in the process when compared to the equivalent of two tangents with a short intervening curve. The fact that the designer may battle to achieve the required radius is, to the driver, a matter of magnificent indifference.

His problems are, to the designer, not a matter of indifference. If he has to provide a 5 000 metre radius curve to achieve an unsuperelevated change of direction and this inexorably takes the road through a sixteenth century cathedral, he may just suspect that his route location is going to attract a certain amount of public participation. How then is he going to ensure that the driver can change direction at his selected speed and in comfort without requiring all of the Karoo to do it in?

2 THE MECHANICS OF THE PROBLEM

Acceleration is normally understood as being an increase in speed. This is however a very limited understanding because deceleration is simply a negative acceleration. Furthermore, in the mathematical scheme of things, acceleration is defined as any change to a vector of translation and, to properly define a vector, two characteristics have to be described being its magnitude and its bearing. Negotiating a curve at unaltered speed is thus also an acceleration. And the acceleration of a moving mass requires a force. The extent of the required force is a function of the speed of the moving object and the radius of the curve it is negotiating.

The force that results in a change of direction is known as centripetal (Afr: Middelpuntsoekend, which is perhaps more expressive). Because every action comes complete with an opposing reaction, a force seeking to resist the change of direction comes into being and this is known as the centrifugal force (Afr: Middelpuntvliegend). If the designer thus wishes to impose a change of direction, this is the force that he must overcome. In terms of systems analysis, the problem definition includes consideration of constraints on the solution. The constraint here is to be found in the goals of transportation including SAFE movement of people and goods. Furthermore, the driver does not only want the assurance that he is safe; he wants to feel safe as well.

There is only one naturally occurring force available without the intervention of the designer and this is the centripetal force which is a product of mass and the coefficient of friction. The coefficient of friction is known as the side force factor and, in theory, this should be independent of speed. In practice, air passing rapidly under a vehicle is compressed and this exerts a lifting force, effectively reducing the mass of the vehicle that can be brought to bear on the development of side force. If the vehicle is moving fast enough, it will take off regardless of actual shape with the shape thereafter deciding if the vehicle will stay airborne once the lifting force created by the compression of air has been removed. The lifting effect is, for convenience built into the side force factor so that this is not entirely a pure coefficient of friction and reduces with increasing speed. Values of side friction that are used in design are given by the relationship

$$f = 0{,}19 - V/1\,600$$

where V = Speed (km/h)

If the vehicle is located on a lateral slope, the vertical force of gravity can be split into vectors, one of which is parallel to the slope and the other at right angles to it. The force parallel to the slope can be represented by a further set of vectors, one vertical and one horizontal, and it is the latter which assists the side force in overcoming the centrifugal force involved in the change of direction. This force, F_g, is expressed as

$$F_g = M\, g \sin\theta \cos\theta$$

The angle, θ, is very small. Its cosine is thus close to 1 and its sine is well approximated by the tan. The expression, g sin θ cos θ, can thus conveniently be replaced by g tan θ and the latter is simply the magnitude of the slope presented in percentage form as a gradient, "e".

If the centrifugal coefficient of acceleration is balanced by the combination of side force and slope coefficients of acceleration, the combination of forces is stable and this stability is recognised in the relationship

$$e + f = V^2 / 127\,R$$

where V = speed (km/h)

R = radius (m)

The designer, armed with the relationships given above, can thus select his design speed and hence calculate the side force coefficient available. Thereafter, he can select a radius of curvature and derive the required superelevation required to balance the forces to zero. Theoretically, and apart from the approximations implicit in the trigonometrical ratios, something approaching a Wall of Death would be perfectly acceptable. Constraints are however always with us.

The most obvious of these is that very steep superelevations would require the vehicle speed to match the design speed to fairly close tolerances. A heavily loaded truck on a steep up grade would obviously have a problem. If the load, in addition to being heavy, is also high there is a distinct possibility that it either would drop its load all over the road or roll, the selection of consequence depending on how well the load was secured.

In the early Sixties, the maximum gradient for use on primary rural roads was 1 : 16 or 6,25 % and the maximum rate of superelevation was 12 %. The combination of these two maxima was found to be adequate to dislodge bales of fodder. The maximum rate of superelevation for rural roads was thereafter reduced to 10 %. Some time later, and for different reasons, the maximum gradient was reduced to 5 % and unsecured bales of fodder are now perfectly safe.

Urban situations are seldom plagued by trucks loaded to great heights with fodder but other restraints come into play. Few urban streets have a total control of access and property access requires that the street should be fairly close to natural ground level at all times. A 10 % superelevation would be inimical to this requirement. Intersections are also closely spaced and it can occur that an intersection may coincide with a curve. Treating vehicles to an adverse superelevation of 10 % is an open invitation to disaster. The distance available to develop superelevation is, in any event, invariably restricted. For this reason, urban superelevation rates are limited desirably to 4 % or to a maximum of 6 %.

Having decided what the maximum allowable value of e is, it is a simple matter to calculate what minimum radius can be used for any given design speed and these values were listed in the previous lecture.

3 SELECTION OF SUPERELEVATION FOR RADII GREATER THAN THE MINIMUM

The values of e and f referred to above are maxima and apply to the minimum radius. For a radius greater than the minimum, it follows that less than maximum values of e and f would be utilised. They thus become semi-independent variables in the sense that they both can be varied but their sum must match the centrifugal coefficient.

There are five recognised methods for distributing superelevation and friction for radii greater than the minimum. These are

- ❑ e and f are increased uniformly and proportionally to the degree of curve
- ❑ f only is used until a further decrease of radius would require the introduction of e
- ❑ e only is used until e_{max} has been reached and then further reductions of radius are balanced by increasing f
- ❑ as for the third method but based on average running speed rather than design speed
- ❑ both e and f are increased in a curvilinear relationship with degree of curve.

The reference to "degree of curve" requires some elucidation. In the good old days of Imperial measure, setting out of roads was based on a 100 foot chain. Degree of curvature was a reference to the magnitude of the central angle subtended by an arc 100 feet long. Degree of curvature is related to radius by virtue of the expression

$$L = R.\theta$$

where L = arc length
R = radius
θ = central angle in radians

With an arc length of 100 feet and the angle, θ, converted to degrees, D, by multiplying by $180/\pi$ (or 57,296) the expression thus becomes

$$D = 5\,729,6 / R$$

South African design practice has opted for the fourth of the methods given above, whereas the American favour the fifth. The South African selection implies that driving at the average running speed, generally about 85 % of design speed, will cause the curve to be negotiated with a "hands off" level of control, which seems to suit most South African drivers.

4 SUPERELEVATION DEVELOPMENT

Discussion so far has purely indicated what the superelevation for a given radius at a selected design speed should be. It is therefore necessary to consider how, from a normal cross-section, this superelevation would be achieved. The usual cross-section of an undivided road has a central high point with the lanes falling away towards the shoulder breakpoints. Superelevation implies a constant crossfall, equal or greater in magnitude than the normal camber, between shoulder breakpoints. This condition can be achieved by holding the one lane edge at a constant height relative to the centreline and rotating the other around the centreline until the camber has been replaced by a cross-fall. Both edges are then rotated around the centreline until the desired extent of superelevation has been achieved.

Rotation, in the case of a two-lane road, is typically around the centreline. However, there is no requirement that this has to be the case and rotation can be around any point of the cross-section that is convenient to the designer.

In the case of a dual carriageway, rotation is often around the inner shoulder breakpoint. If the selected cross-section includes a crossfall from the inner edge of the inside lane towards the median, the point of rotation would actually be somewhere in the air and on the extension of the crossfall across the lanes to a point above the inner shoulder edge.

For purposes of calculation, superelevation development is split into two components being crown runoff and superelevation runoff. Crown runoff is the distance required to rotate the outside lane to the point where the lane edge and the centreline are at a common height, ie the adverse camber has been removed. Superelevation runoff is the distance needed to rotate the outside lane from level to full superelevation.

The rate of rotation is measured by the relative slope between the carriageway edge and the axis of rotation. The relative slope factors quoted in the table below have been found in practice to give acceptable lengths of run-off.

The length of run-off is thus calculated as

$$L = w.e.s.l/100$$

where
- w = lane width
- e = extent of superelevation
- s = slope factor (reciprocal of relative slope)
- l = lane factor

and the last mentioned factor calls for explanation

Table A3.1: Relative Slope Factor	
Design Speed (km/h)	**Relative slope factor 1 in**
40	140
60	170
80	200
100	230
120	260

Where the rotated surface is wider than one lane, the additional width would automatically result in an increase in the distance required to achieve the development of superelevation. And, very often, this additional distance is simply not available. In the case of undivided roads, runoff length is calculated on the basis of that required for a two-lane road and a lane correction factor applied. Lane factors are as given below.

The length of crown run-off is calculated using the same formula but with the value of superelevation, e, replaced by the normal camber, typically 2 %.

Where the superelevation required for a particular radius is relatively low, blind adherence to the recommended relative slopes would result in a short developmental section that would inevitably create the appearance of a kink in the road edge. To avoid this effect, the runoff length should be as given below.

Table A3.2: Lane Factors For Superelevation Run-Off			
Cross-section	**Median width (m)**	**Number of lanes**	**Lane Factor**
Undivided		2	1,0
	-	3	1,2
	-	4	1,5
Divided	Less than 4,6 m	2	1,5
		3	2,0
	Between 4,6 and 12,2 m	2	1,0 or 1,5
		3	1,2 or 2,0
	Greater than 12,2m	2	1,0
		3	1,2

Table A3.3: Minimum Length Of Superelevation Run-Off For Two-Lane Roads	
Design Speed (km/h)	**Run-off (m)**
60	40
80	50
100	60
120	70

While a preferred minimum length of superelevation run-off is quoted, no maximum lengths are suggested. The designer should however be aware of the fact that a long run-off might cause drainage problems at the commencement of the run-off section.

A preferred method for avoiding kinked road edges is to grade the road edges separately. This method is not without its complications when the axis of rotation is on any line other than a carriageway edge because of the possibility of creating a surface so warped that it defies the skill of the contractor to build it.

5 LOCATION OF SUPERELEVATION RUN-OFF

The location of the position of the superelevation run-off is somewhat of a compromise. The driver should have the full superelevation on reaching the start of the curve. On the other hand, having the full superelevation while still on the tangent preceding the curve is operationally awkward, to say the least. The compromise achieved is to have two-thirds of the runoff on the tangent and one-third on the curve. This more-or-less matches the transitional path that the vehicle follows in moving from the tangent to the curve.

6 TRANSITION CURVES

Transition curves are much beloved of town and regional planners who draw them on paper and disliked by surveyors who have to set them out. In fact, setting out a transition curve is not all that difficult. The transition curve is a spiral that commences at infinite radius (ie straight) and terminates with a radius matching that of the adjacent circular curve.

Transition curves have distinct advantages:

- ❑ the curve leads the driver into the circular curve and causes increases and decreases in centrifugal force to occur gradually.
- ❑ without this leading action, the driver typically has already entered the curve before realising it and thus follows a circular path tighter than that of the selected curve radius. Under limiting conditions this is not a safe practice.
- ❑ the length of the spiral curve is used for superelevation run-off.

- the length of the spiral curve can be used to incorporate the transition to a wider section of the roadway around the curve. (Obviously it should not be used to incorporate a decrease in roadway width. The driver needs to be warned about an impending lane- drop and hiding it on a transition, doesn't do him any favours at all. Lane drops should be out there where they can be readily perceived and reacted to.)
- the transition enhances the appearance of the highway by removing the noticeable breaks at the beginning and end of circular curves, because, without a transition, there is an instantaneous change from one radius (infinite) to another (relatively small- otherwise the thought of transitions would not have been entertained)

As a rule of thumb, transition curves should be considered if the radius selected is such that the superelevation required is about 80 % of whatever e_{max} has been selected. This approach addresses the problem of the driver who might wake up too late to the fact that he is on a minimum radius curve so that the path followed is at a sub-minimum radius hence requiring more superelevation than has actually been provided. This, of course, is also a sound argument in favour of attempting to avoid such critical radii.

6.1 Form of transition

Various curves can be employed as transitions. Whatever form of curve is selected, it should satisfy the conditions that

- it is tangential to the straight
- its curvature should be zero (ie infinite radius) on the straight
- the curvature should increase (ie radius decrease) along the transition at the same rate that e increases
- its length should be such that at its junction with the circular curve, full superelevation has been attained
- it should join the circular arc tangentially
- the radius of the transition curve at its end should be the same as that of the circular curve

Curves which can be used as transitions are the Euler Spiral, the Lemniscate and Froude's Spiral otherwise known as the cubic parabola. The last mentioned achieves a maximum value and then flattens out again so it is not a true spiral. The lemniscate requires an unacceptable length of arc to achieve the desired radius and the Euler Spiral is the preferred form.

The first two and last two conditions are definite but the rate of change of radius and the length of the curve depend on the rate at which superelevation is introduced. In practice, superelevation is developed at a uniform rate, specifically as expressed by the relative slope factor, so that the radius of the transition curve at any point is proportional to its distance from the origin.

6.2 Setting out transition curves

If a circular arc were to be set out between two tangents, it should be clear that any spiral commencing from either of those straights can never meet up with the circular arc. Visualise the situation rather from the other side. A spiral that commences at a predetermined radius and bearing will inexorably wind out from the circular arc. That is, after all, its function. Why would one otherwise have the thing?

In order therefore to fit the transitions between a selected curve radius and the tangents, it is usual to move the curve back from the point of intersection along the line connecting the point of intersection to the centre of the circle. This means fitting the actual circular curve between two new tangents parallel to the old ones but shifted from them by an amount, s, known as the *shift*. The new tangents are thus referred to as the *shift tangents*.

The value of the shift is given by

$$s = L^2 / 24R$$

where L = selected length of transition curve

R = radius of circular curve, ie the final radius of the spiral

It is useful to have a starting point from which to set out the transition and this is found by calculating the tangent length based on the radius of the circular curve plus the shift. This is given by the familiar ratio

$$T = R \tan \theta/2$$

The value of R that is applied is the radius of the circular curve plus the shift so that

$$T = (R + s) \tan \theta/2$$

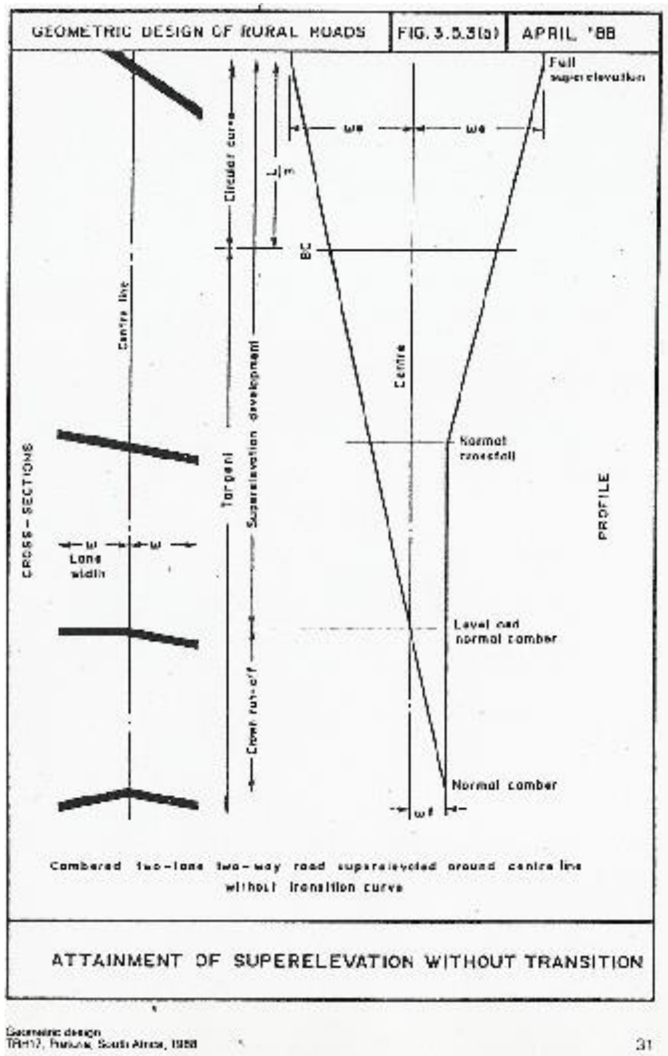
This distance, T, by virtue of the geometry of the situation, will serve to locate a point which is halfway along the transition so the final value of T that is used is

$$T = (R + s) \tan \theta/2 + L/2$$

The most convenient way to set out the spiral is by deflection angles and chords. The deflection angle for any particular chord length is given as

$$\alpha = l^2 / 6RL$$

which is very handy except that α happens to be expressed in radians so that the conversion factor of 57,246 has to be applied. The value of chord length, l, can be selected as anything convenient. For example, it may be decided to develop the superelevation over a distance of 100 m, in which case a chord length of 20 metres would be considered convenient.
Can it be that transition curves are not so terrifying after all?


GEOMETRIC DESIGN OF RURAL ROADS
FIG. 3.5.3(a)
APRIL '88
Full superelevation
Circular curve
BC
Centre line
Centre
Tangent
Superelevation development
Normal crossfall
CROSS-SECTIONS
PROFILE
Lane width
Level and normal camber
Crown run-off
Normal camber
Cambered two-lane two-way road superelevated around centre line without transition curve
ATTAINMENT OF SUPERELEVATION WITHOUT TRANSITION
Geometric design
TRH17, Pretoria, South Africa, 1988
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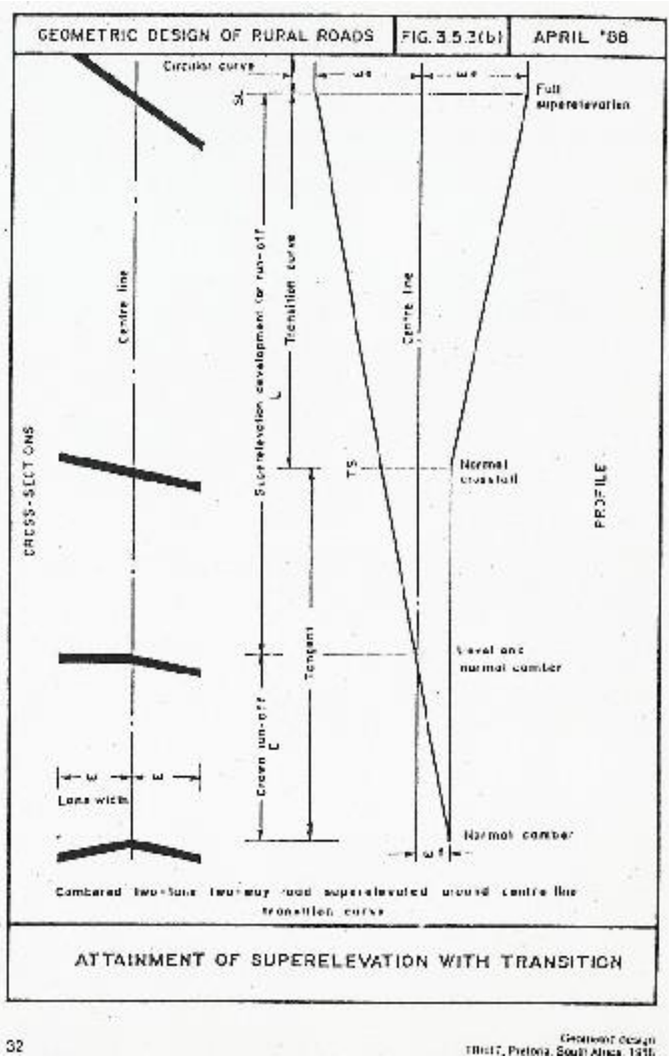
GEOMETRIC DESIGN OF RURAL ROADS
FIG. 3.5.3(b)
APRIL '88
Circular curve
Full superelevation
Centre line
Superelevation development for run-off
Transition curve
Normal crossfall
TS
CROSS-SECTIONS
PROFILE
Tangent
Level and normal camber
Crown run-off
Lane width
Normal camber
Cambered two-lane two-way road superelevated around centre line transition curve
ATTAINMENT OF SUPERELEVATION WITH TRANSITION
32
Geometric design
TRH17, Pretoria, South Africa, 1988