SESSION A   ASPECTS OF DESIGN

LECTURE A5 - VERTICAL ALIGNMENT– SIGHT DISTANCE & CURVES   PRESENTER : CD SCOTT

1 INTRODUCTION

As in the case of the horizontal alignment, the vertical alignment is a sequence of tangents and curves. Tangents are referred to as *"grades"* and the value of the slope at which they occur is the *"gradient"*. Curves can be circular as in European practice or elliptical although the parabola is preferred in the United States. South African practice derives principally from American precedents; hence the local preference is also for the parabola.

In practice, there is little to choose between the circular and the parabolic curve because with a given curve length and specified gradients on either side of it, the difference in levels achieved by using the one in preference to the other are almost unplottable.

Furthermore, once the levels have been calculated with painstaking accuracy to the nearest micromillimetre, the man, responsible for assuring that they get transferred to the base course, is sitting on a grader and, consequently, has an eye height closer to three metres than to two. He will positively guarantee that any differences between the circular curve and the parabola are totally undetectable.

2 THE SHAPE OF THE VERTICAL CURVE

In theory, the parabola is vastly to be preferred over the circular curve. The horizontal circular curve treats the driver to a constant rate of change of bearing and the parabola provides a constant rate of change of gradient, ie:

The rate of change in gradient, $\frac{dy}{dx}$, is constant, or, written mathematically

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = a = \text{Constant}$$

By integration, the gradient is found:

$$\frac{dy}{dx} = ax + b$$

Considering boundary conditions:

Gradient at beginning of curve: $\frac{dy}{dx} = G_b$, at x = 0

Gradient at end of curve: $\frac{dy}{dx} = G_e$, at x = L

Substituting:

$\therefore b = G_b$ $\therefore G_e = a.L + G_b$

Thus: $a = \frac{G_e - G_b}{L} = \frac{A}{L}$ the difference in gradient / length of curve, or, $\frac{1}{K}$

Where $K = \frac{L}{A}$, the length of curve / unit change in gradient, or *sharpness* of curve

And $L = K.A$, the length of curve required for specified K and relevant A

By integration, again, the height, y, along the curve is given by:

$$y = \frac{a.x^2}{2} + b.x + c$$ which is a parabolic curve.

By substitution:

$$= \frac{1}{2}.\left(\frac{G_e - G_b}{L}\right).x^2 + G_b.x + c$$

With boundary condition:

At beginning of curve, x = 0, the height, y = H_b, and therefore c = H_b

Thus, the height at any point along the curve is given by:

$$y = \frac{1}{2}.\left(\frac{G_e - G_b}{L}\right).x^2 + G_b.x + H_b$$

or

$$y = \frac{1}{2}.\left(\frac{1}{K}\right).x^2 + G_b.x + H_b$$ in metres

$$y = \frac{x^2}{200.K} + \frac{G_b}{100}.x + H_b$$ Gradients in %

A being the algebraic difference in gradients, or the total change in gradient across the length of the vertical curve and K being equal to L/A. K, described in words, is the distance required for a unit change of gradient. Obviously it does not matter how the gradient is described; whether as a percentage or as a slope of 1 in x (metres or curved Viennas); the length of the curve will be in the same unit as that adopted for K. Equally obviously, percentage gradients, being the height difference (in feet or metres) achieved across a horizontal distance of 100 feet or metres, are the most convenient to work with.

3 THE CALCULATION OF VERTICAL CURVES

In an era when the handheld calculator exceeds the computational ability of the main frame of thirty years ago, it doesn't really matter much how one sets about calculating the levels along a vertical curve. It is however useful to have some idea of how one should set about carrying out the exercise.

There are two fundamental forms of calculation to be considered. These are the Offset Method and the other the Gradient Method.

The Offset Method calculates a level on the extension of the first grade beyond the start of the vertical curve, the BVC, and then goes on to calculate the height difference between this point and the alignment to finally achieve the gradeline level required. The attraction of this method is that it is possible to very easily calculate a gradeline level at any point on the curve as opposed to being locked into calculating levels at fixed intervals along the road.

The principal attraction of the Gradient Method is that it is self-checking. It involves two distinct steps. The first step is calculation of the gradient between all the successive pairs of points on the road and is checked by the fact that the gradient at the end of curve (EVC) is in fact equal to the gradient selected for that grade. The next step uses the height of the preceding point and the gradient to the succeeding point to establish its height. This process is continued until the EVC is reached and is checked by the fact that the height of the EVC calculated along the curve should match that calculated from the vertical point of intersection, the VPI. This method had its finest flowering when calculation was manual or by use of the more sophisticated Facit Mechanical Calculator. Being able to easily check the correctness of your work was a distinct asset for any system of calculation.

Computer programs supposedly do not need all these checks and the Offset Method is the one generally applied.

This method goes directly to the intrinsic equation of the parabola by calculating

$$y = ax^2 + bx + c$$

where y = the elevation of any point along the curve

x = horizontal distance beyond the BVC

A small aside: Height is a difference ie the height of a point on the curve above the BVC. Elevation is an absolute in that it refers to a datum external to the alignment with the datum being something like MSL or LWOST.

It is necessary in the first instance to calculate the parameters, a, b and c, of the curve.

If $x = 0$ then $y = c$

so that the parameter c is the elevation of the BVC

Also for $x = 0$, $dy/dx = G_b$

and parameter G_b is thus the initial gradient from which the vertical curve springs.

The second derivative, which is the actual descriptor of the shape of the curve, has already been discussed and this is selected by the designer in terms of the particular curve he elects to use. Selection is typically in terms of a value of K which is equal to L/A with A being the algebraic difference in gradients of the tangents on either side of the curve. As has been demonstrated,

$$a = 1/2K$$

The equation can thus be rewritten as

$$y = (1/200K).\ x^2 + (G_b / 100).\ x + H_{BVC}$$

and this demonstrates the origin of the name Offset Method. The last two terms provide the elevation along the extension of the first tangent at the desired point x and the first term represents an offset either above or below this point on the tangent.

As a precursor to calculation of the vertical curve, the designer must decide on the vertical location of the adjacent grades and this refers to their gradients as well as the elevation of a point along them, with the fixed points usually being the preceding and succeeding VPI's. In days of yore, it used to be worthwhile putting a little effort into ensuring that the VPI. BVC and EVC fell on full stake values because this reduced the pain of manual calculation. This is no longer a reason but it is still practical to have the BVC and EVC falling on full stake values in terms of the convenience of setting out and construction. The designer will have selected a K-value of vertical curve to use and this, in association with the values of the gradients, will determine the length of the curve from which it is easy to calculate the elevations of the BVC and EVC. He should also have some idea of what the stake interval should be because these are the points at which the surveyor will need to know what the gradeline height is.

The designer thus provides input of

S	=	VPI stake value
H	=	VPI elevation
G_b	=	Initial gradient
G_e	=	Final gradient
K	=	K-value of selected curve

or

L	=	Curve length

4 PLOTTING OF VERTICAL CURVES

The vertical alignment is usually drawn to a distorted scale of 1:10 between the vertical and the horizontal. The scale factor is simply a matter of convenience because gradients are small, typically less than 5 %., and differences in successive gradients not easy to see when drawn to a natural scale.

Vertical curves are usually drawn using railway curves which are, in fact, circular but, because of the distortion, are a reasonable approximation of parabolas. The radius of railway curve in centimetres that is required to match a particular K-value can be calculated from the relationship

$$R_c = 100.K.Vert/Hor^2 \quad + \quad 100/(6,5.\ Hor)$$

where Vert = reciprocal of vertical scale
Hor = reciprocal of horizontal scale

As a rough rule of thumb, the radius of the railway curve in centimetres is equal to the K-value of the vertical curve when the distortion is 1:10. While still on thumb-like rules, this applies to the middle third of the curve. The outside thirds can be approximated by using a longer radius railway curve between the plotted position of the BVC or EVC and the already drawn section of the curve. The outside curves can have anything up to double the radius of the inner curve.

It sometimes happens that the curve being drawn is so long that it cannot be drawn with a single template. A useful device then is to draw a line between the midpoints of the tangents connecting the BVC and EVC to the PI. The midpoint of this line falls on the vertical curve and the line is also tangential to the curve. The process can be repeated ad nauseaum until sufficient points are obtained to enable the fitting of the curve.

5 SIGHT DISTANCE AND CURVATURE

The minimum curvature that may be adopted is based on stopping sight distance for the selected design speed of the road in the case of a crest curve. Reference here is to a grazing ray from an eye height of 1,05 m to an object height of 0,15 m. Minimum curvature in the case of a sag curve is based on headlight illumination distance where the assumption is that the headlight is mounted at a height of 0,6 m above the centreline of the road, and the spread of useful light has a divergence angle of 1^{o} above the horizontal.

The value of minimum curvature that can be adopted to provide adequate sight distance is derived from the parabolic formula, as follows:

$$y = ax^2 + bx + c$$
$$= 1/2K\ x^2 + bx + c$$

Measuring from the point at which the grazing ray touches the vertical curve it follows that c = 0, and it is also convenient to assume a horizontal ray, ie b = 0, so that

$$y = 1/2K\ x^2$$

Two values of y are of interest, being the driver eye height, h_1, and the object height, h_2. Two values of x can be calculated, x_1 corresponding to h_1 and x_2 corresponding to h_2. The sum of these two values of x is the sight distance required, so that

$$S = x_1 + x_2$$

$$= (2K.h_1)^{0,5} + (2K.h_2)^{0,5}$$

from which it follows that

$$K = \frac{S^2}{(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

Minimum K-values compatible with stopping sight distance are given in the table below.

Table A5.1: Minimum values of k for vertical curves

Design speed (km/h)	**Crest curves**	**Sag curves**
40	6	8
50	11	12
60	16	16
70	23	20
80	33	25
90	46	31
100	60	36
110	81	43
120	110	52
130	133	57
140	163	64