Assessment and Analysis of Test Data

Presented by SARF

Presenter:
Ron Berkers

1

Module D
Judgement of Compliance

2

Quality Management Scheme:

- The information available on which to base a decision whether a product should be accepted or rejected is usually obtained from sampling a population.
- Even the highest standard of product sampling and testing cannot – and does not – provide flawless.
- There is always uncertainty about the correct decision due to sampling and testing inaccuracies and the inherent variation of the product being assessed.
- In judgement schemes no measurement is certain and consequently acceptance procedures are not always fair to both the supplier (contractor) and the client.
- Consequently statistical quality assurance procedures should be applied correctly and every precaution taken to minimise the risk of wrongful assessment of quality.

- An important aspect of the quality control is the detection of both random and systematic errors.
- For the detection of error type, as well as the quantification of the errors, statistical treatment of data is indispensable.
- In analytical work, the most important common operation is the comparison of data, or sets of data, to quantify accuracy (bias) and precision.
- The mere collection of facts will not constitute statistics unless they have been subjected to formal enquiry.
- Statistical methods should be used intelligently and carefully as their misuse will almost certainly lead to unsatisfactory and even hazardous outcomes.
- False conclusions will follow if data collected is incomplete and unreliable.

4

Judgement Schemes:

Judgment schemes can be divided into two classes:

Quality Control (QC)
Quality Assurance (QA).

Quality:

- Quality is determined by the users of product, clients or customers.
- If the specification does not reflect the true quality requirements, the product's quality cannot be guaranteed.

  For example, the parameters for a heating vessel should cover not only the material and dimensions but operating, environmental, safety, reliability and maintainability requirements.

- Managing quality on a project requires a clear understanding of the specific quality expectations of the customer followed by the implementation of a proactive plan to meet those expectations.
- The "proactive plan" contains a number of elements - the most important of which are the QC and QA activities that need to be performed.

5

Quality Control:

- Quality control (QC) is a procedure or set of procedures intended to ensure that a manufactured product or performed service adheres to a defined set of quality criteria or meets the requirements of the client or customer.
- Quality control is the more traditional way that businesses have used to manage quality.
- Quality control is concerned with checking and reviewing work that has been done.
- Quality control emphasises testing of products to uncover defects, and reporting to management who make the decision to allow or deny the release.

6

Quality Assurance:

- QA is defined as a procedure or set of procedures intended to ensure that a product or service under development (before work is complete, as opposed to afterwards) meets specified requirements.
- QA can be described as the systematic monitoring and evaluation of the various aspects of a project, service or facility to maximize the probability that minimum standards of quality are being attained by the production process.
- Two principles included in QA are: "Fit for purpose", the product should be suitable for the intended purpose; and "Right first time", mistakes should be eliminated.
- QA includes regulation of the quality of raw materials, assemblies, products and components, services related to production, and management, production and inspection processes.

7

Statistical Population:

A population is any entire collection of people, animals, plants or things from which we may collect data. It is the entire group we are interested in, which we wish to describe or draw conclusions about.

Sample:

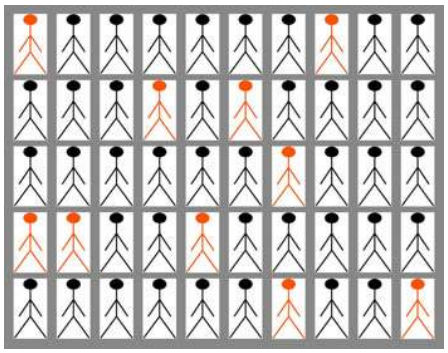
- A sample is a group of units selected from a larger group (the population).
- By studying the sample it is hoped to draw valid conclusions about the larger group.
- A sample is generally selected for study because the population is too large to study in its entirety.
- The sample should be representative of the general population.

8

Sampling

Random Sample:

- Random sampling is the simplest method of probability sampling.
- Within a particular study population everyone has an equal chance of inclusion in the sample.
- It is considered 'fair' and therefore allows findings to be generalized to the whole population from which the sample was taken.

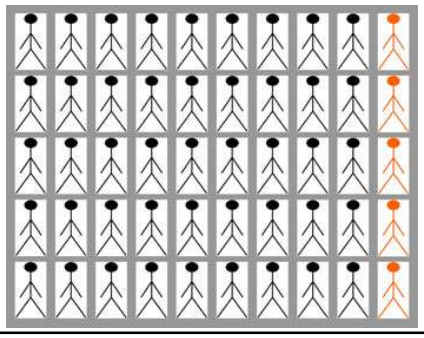


9

Sampling

Systematic Sample:

- In systematic sampling, individuals are chosen at *regular intervals* using a sampling frame to help you do this.
- Systematic sampling does not give every individual in a population the same chance of selection.
- It is usually easier and quicker to obtain a systematic sample than a random one.
- It is important to ensure that the sampling method does not introduce a consistent bias into the sample.

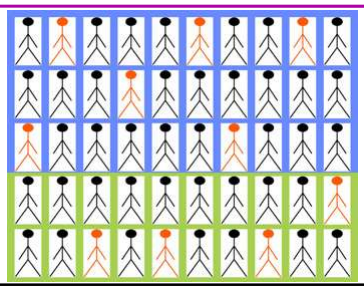


10

Sampling

Stratified Sample:

- Stratified random sampling involves dividing your population into various subgroups and then taking a simple random sample within each group.
- This will ensure that your sample represents key subgroups of the population.
- Representation of the subgroups can be proportionate or disproportionate.
- The basic purpose of stratification as compared to simple random sampling is to obtain a reduction in sampling error or, synonymously, an increase in precision.



11

Sampling

Click picture

12

Sample Size:

- How large should a sample be in any specific situation?
- If a sample is used which is larger than necessary, resources are wasted.
- If the sample is smaller than required, the risk of wrongful decisions may be increased and the objectives of the analysis not be achieved.
- Two questions must be answered to specify the required sample size:
  What degree of precision is desired?
  How probable must it be so that the desired precision will be obtained?
- The greater the degree of desired precision, the larger will be the necessary sample size.
- Also, the greater the probability specified for obtaining the desired precision, the larger will be required sample size.

13

Sample size n:

$$n = \left(\frac{z_\alpha \cdot \sigma}{\varepsilon}\right)^2$$

Where:

n	is the number of observations or measurements required for a statistical sample aiming for a certain precision.
σ	is the population standard deviation.
ε	is the required precision aimed at.
z_α	is a standardized factor for a confidence level of α .

For α = 5% it follows:

$$z_\alpha = z_{0,05} = \begin{cases} 1{,}65 \text{ if a one-sided case} \\ 1{,}96 \text{ if a two-sided case} \end{cases}$$

14

Example:

Determine the statistical sample size for an investigation to determine the layer compaction with a confidence interval of 95%. The precision error required should be equal or less than 0,5% . The assigned standard deviation is 1,8%.

Solution:

This is a one-sided case, because only a single limit for compaction, i.e. minimum, is specified.

$$z_{0,05} = 1,65$$

$$n = \left(\frac{(1,65)\cdot(1,8)}{0,5}\right)^2$$

$$n = 35$$

15

Example:

Determine the statistical sample size for determining Maximum Dry Density of a certain type of material. The precision error in estimating the mean density shall be less than 10 kg/m^3 with a confidence interval of 95%. The assigned standard deviation for the material is equal to 25 kg/m^3 .

Solution:

This is a two-sided case, because density and water content are specified.

$$z_{\alpha/2=0,025} = 1,96$$

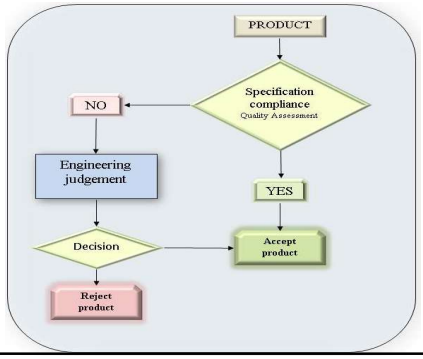
$$n = \left(\frac{(1,96)\cdot(25)}{10}\right)^2$$

$$n = 24$$

16

A Quality Assessment Model

- The information for a decision whether a property (product) is to be accepted as complying with a specific requirement or, otherwise, rejected is inevitably based on tests on samples of a population.
- Since this system does not provide comprehensive information about the product, quality assessment models have been developed to manage the risk of wrongful actions based on the outcomes of limited information.
- The quality assessment model used in the road construction industry can be schematically pictured as follows:



17

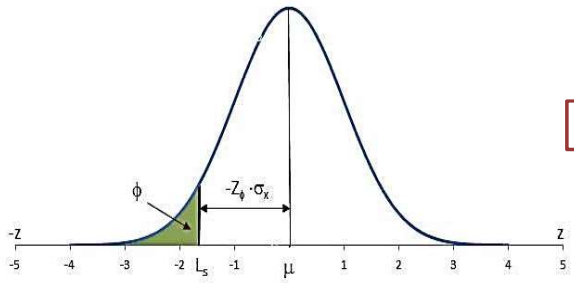
A Quality Assessment Model

- Almost all of the properties of road building materials which we assess for acceptance have near-normal distribution.
- The normal distribution is fully described by the two population parameters, i.e. the population mean (μ) and the population standard deviation (σ).
- The problem that we have is that these two parameters are most of time unknown and must be estimated from the sample statistics.

18

A Quality Assessment Model

For a normally distributed population where a single lower specification acceptance limit L_s is specified:



$$\boxed{\mu = L_s + z_\varphi \cdot \sigma}$$

Where:

φ (phi)	is the percentage defectives allowed in a normally distributed population of a certain property.
z_φ	is the standardised constant for a normal distribution which is related to φ.
L_s	the lower specification limit for individual property test values or measurements.
σ	is the standard deviation of the population.
μ	is the mean of the population

19

A Quality Assessment Model

Example:

By analysing a very large number of compaction values an analyst determined the following:

$$\mu = 99{,}5$$
$$\sigma = 1{,}00$$

What is the percentage of the total defectives (φ) allowed if the required limit for an acceptable compaction specification is L_S = 98,7?

Solution:

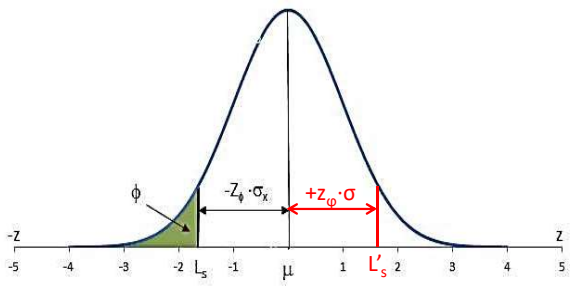
$$\mu = L_s + z_\varphi \cdot \sigma \longrightarrow z_\varphi = \frac{\mu - L_s}{\sigma} = \frac{99{,}5 - 98{,}7}{1{,}00} = 0{,}800$$

Area φ = 0,5 – area z_φ = 0,5 – 0,288 = 0,212 (table 8)

Which is 21,2 %

20

A Quality Assessment Model



For a single upper limit specification is the acceptance limit L_s':

$$\mu = L_s' - z_\varphi \cdot \sigma$$

For dual specification limits the value of φ is normally relevant to both upper and lower specification limits and the following has to be applied simultaneously:

$$\mu = L_s + z_\varphi \cdot \sigma$$
$$\mu = L_s' - z_\varphi \cdot \sigma$$

21

Statistical Sample

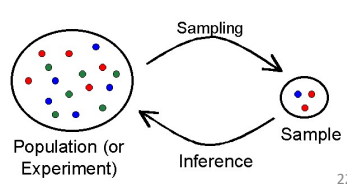
Statistical sample:

- **Consists of two variables:**

  sample mean, $\bar{x}$
  sample standard deviation, *s*

- **and a constant, sample size *n*.**

- **as the sample size increases the variation in sample means and standard deviations decrease.**



22

Acceptance Criterion

Terms used:

- L_A = lower acceptance limit for the sample mean
  L_R = the rejection limit for the sample mean.
  L_S = single lower acceptance limit
  αA = the risk when an acceptable product is rejected (contractor's risk).

- The risk to the contractor is called the α-risk:

  If the confidence interval is 95%, then the α-risk is 5% or 0.05.

  For example, there is a 5% chance that a part has been determined defective when it actually is not. One has observed or made a decision that a difference exists but there really is none.

- The risk to the client is called the β-risk:

  β-risk is the risk that the decision will be made that the part is not defective when it really is. In other words, when the decision is made that a difference does not exist when there actually is

23

Judgement Limits

For Compliance Limits:

Lower acceptance limit $L_A = L_s + F_A \cdot s$
Upper acceptance limit $L'_A = L_s - F_A \cdot s$
Lower rejection limit $L_R = L'_s + F_R \cdot s$
Upper rejection limit $L'_R = L'_s - F_R \cdot s$

F_A can be obtained from table 14(a) for single limit specification (α=5%) and table 14(c) for double limit specification (α=5%).

F_R can be obtained from table 14(b) for single limit specification (α=1%) and table 14(d) for double limit specification (α=1%).

24

Judgement Limits

Example:

The following sets of bitumen contents were obtained from a sections of asphalt, manufactured and laid according to a certain specification, given below:

Specification:

Bitumen content	5,50
Tolerance	0,25
Lower limit L_s	5,25
Upper limit L'_s	5,75
Φ - value	12%

25

Judgement Limits

Example (Continued):

Bitumen Content Test Results:

Lot	I
X_1	5,3
X_2	5,3
X_3	5,4
X_4	5,2
X_5	5,1

Statistics:

- $\bar{x}$ = sample mean = (5,3+5,3+5,4+5,2+5,1) / 5 = 5,26
- $s = \sqrt{((5,3-5,26)^2+(5,3-5,26)^2+(5,4-5,26)^2+(I5,2-5,26I)^2+(I5,1-5,26I)^2)/5-1}$
  $= \sqrt{(0,04^2+0,04^2+0,14^2+0,06^2+0,16^2) / 4}$
  $= 0,114$
- n = 5
- F_A value (α=0,05;n=5;ϕ=12%) = 0,294 (out of table 14(c))
- F_R value (α=0,01;n=5;ϕ=12%) = 0,024 (out of table 14(d))

26

Judgement Limits

Example (Continued):

$\bar{x}$	5,260
s	0,114
n	5
F_A value	0,294
F_R value	0,024

Compliance Limits:

- Lower acceptance limit $L_A = L_s + F_A \cdot s$
  $= 5{,}25 + (0{,}294 \times 0{,}114)$
  $= 5{,}284$
- Upper acceptance limit $L'_A = L_s - F_A \cdot s$
  $= 5{,}75 - (0{,}294 \times 0{,}114)$
  $= 5{,}716$
- Lower rejection limit $L_R = L'_s + F_R \cdot s$
  $= 5{,}25 + (0{,}024 \times 0{,}114)$
  $= 5{,}253$
- Upper rejection limit $L'_R = L'_s - F_R \cdot s$
  $= 5{,}75 - (0{,}024 \times 0{,}114)$
  $= 5{,}745$

- The test results are accepted conditionally (mean = 5,260 lies in transition zone ; 5,710 = Accept ; 5,250 = Reject).

27

Resubmission of lots

In order to establish analytically whether there is a significant difference between two sets of values, the following procedure is normally followed:

1. The variance of the two samples is compared for significant dissimilarity.
2. Where the difference in variance is deemed to be insignificant, i.e. occurring by chance, the means of the sample sets will be compared.

Variance:

1. F-value is calculated with help of : $F = \frac{S_1^2}{S_2^2}$

   Where

   s_1 is the greater value of the two sample standard deviations
   s_2 is the lesser valued of the two sample standard deviations

2. The value of $F_{0,05\ v1;v2}$ is obtained from Table 10.

If $F > F_{0,05\ v1;v2}$ a significant difference is deemed to occur between the two sets of test data.
If $F < F_{0,05\ v1;v2}$ the difference is not necessarily significant and requires that a t-test be conducted.

28

Resubmission of lots

Means:

1. The t-test is conducted as follows:

$$s = \sqrt{\frac{(n_a - 1)s_a^2 + (n_b - 1)s_b^2}{n_a + n_b - 2}}$$

Where the suffixes a and b refer to the two sets of data being considered.

2. t is calculated as follows:

$$t = |\overline{x}_a - \overline{x_b}| / s \cdot \sqrt{1/n_a + 1/n_b}$$

3. The value of $t_{0,05;v}$ is obtained from Table 9 where

$$v = n_a + n_b - 2$$

Where $t > t_{0,05;v}$ a significant difference between the sets of results is indicated.
Where $t < t_{0,05;v}$ the difference in means is not significant.

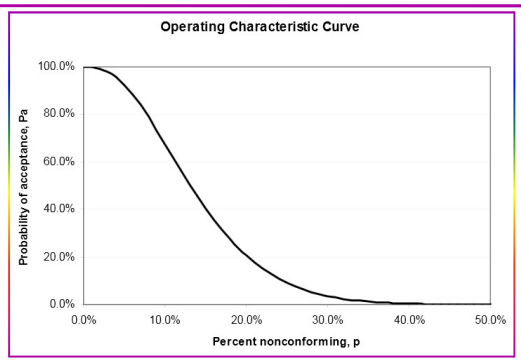
29

The Operating Characteristic Curve

The OC Curve:

The Operating Characteristic (OC) curve describes the probability of accepting a lot as a function of the lot's quality.

The figure below shows a typical OC Curve:



As the lot percent nonconforming increases, the probability of acceptance decreases.

30

Process Control Charts

- The variability in key quality characteristics usually arises from three sources:

1. improperly adjusted machines or equipment;
2. operator errors; and
3. defective raw materials

- The above sources of variability are not part of the chance cause pattern and are referred to as assignable causes.

- A major objective of statistical process control is to quickly detect the occurrence of assignable causes so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured.

- The control chart is an online process-monitoring technique widely used for this purpose.

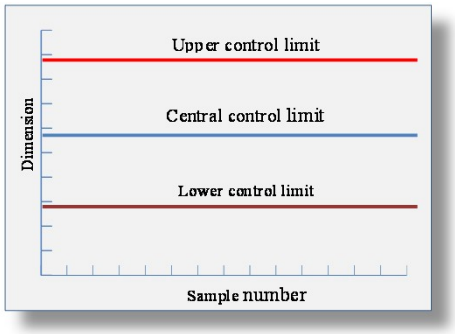
- The control chart can also provide information that is useful in improving the process.

31

Process Control Charts

The process control chart:

- A typical control chart is shown below:

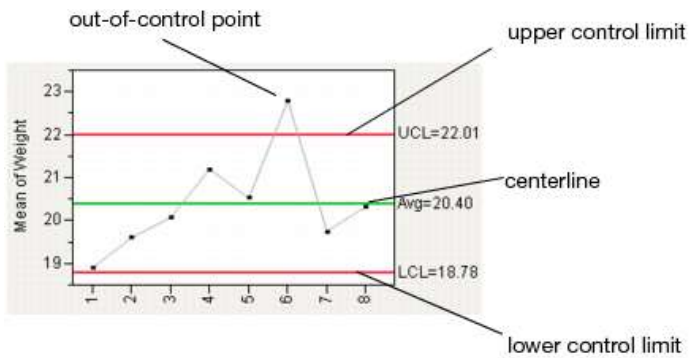


- The control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time.

32

Process Control Charts

- The chart contains a center line (CL) that represents the average value of the quality characteristic.
- Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL), are chosen so that if the process is in control (no assignable causes, only chance cause variations), nearly all of the sample points will fall between them.



33

Process Control Charts

$\overline{X}$ - Chart:

- For a known μ (population mean) and σ (standard deviation):

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$CL = \mu$$

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

- If μ and σ are not known (see table next page):

$$UCL = \overline{X} + A_2\overline{R}$$

$$CL = \overline{X}$$

$$LCL = \overline{X} - A_2\overline{R}$$

34

Process Control Charts

Table 1: Control Chart Constants

n	A_2	D_3	D_4
2	1.880	0	3.267
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.283	1.717
13	0.249	0.307	1.693
14	0.235	0.328	1.672
15	0.223	0.347	1.653

35

Process Control Charts

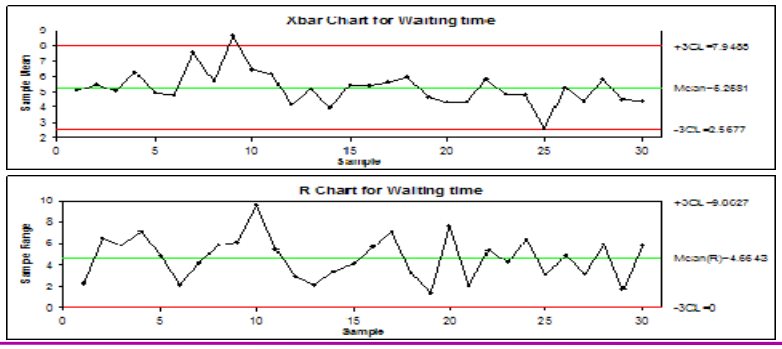
R – Chart:

The center line and upper and lower control limits for R chart are:

$$UCL = D_4\bar{R}$$

$$CL = \bar{R}$$

$$LCL = D_3\bar{R}$$



36

Process Control Charts

Example:

A component part for a jet aircraft engine is manufactured by a casting process. The vane opening on this casting is an important functional parameter of the part.

The following table presents 20 samples of five results each:

n	X_1	X_2	X_3	X_4	X_5	Mean	Range
1	33	29	31	32	33	31,6	4
2	33	31	35	37	31	33,4	6
3	35	37	33	34	36	35,0	4
4	30	31	33	34	33	32,2	4
5	33	34	35	33	34	33,8	2
6	38	37	39	40	38	38,4	3
7	30	31	32	34	31	31,6	4
8	29	39	38	39	39	36,8	10
9	28	33	35	36	43	35,0	15
10	38	33	32	35	32	34,0	6
11	28	30	28	32	31	29,8	4
12	31	35	35	35	34	34,0	4
13	27	32	34	35	37	33,0	10
14	33	33	35	37	36	34,8	4
15	35	37	32	35	39	35,6	7
16	33	33	27	31	30	30,8	6
17	35	34	34	30	32	33,0	5
18	32	33	30	30	33	31,6	3
19	25	27	34	27	28	28,2	9
20	35	35	36	33	30	33,8	6

37

Process Control Charts

Example (Continued):

Mean of means: $\overline{X} = 33,3$
Mean of ranges: $\overline{R} = 5,8$

Out of table 1 for n=5

$$A_2 = 0,577$$

For the $\overline{X}$ - Chart:

$$UCL = \overline{X} + A_2\overline{R}$$
$$= 33,3 + 0,577 \times 5,8$$
$$= 36,65$$

$$LCL = \overline{X} - A_2\overline{R}$$
$$= 33,3 - 0,577 \times 5,8$$
$$= 29,59$$

38

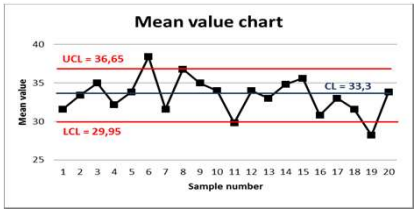
Process Control Charts

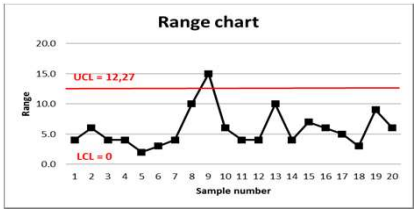
Example (Continued):

For the $\overline{R}$ - Chart:

$$UCL = D_4\overline{R}$$
$$= 2{,}114 \times 5{,}8$$
$$= 12{,}27$$

$$LCL = D_3\overline{R}$$
$$= 0 \times 5{,}8$$
$$= 0$$





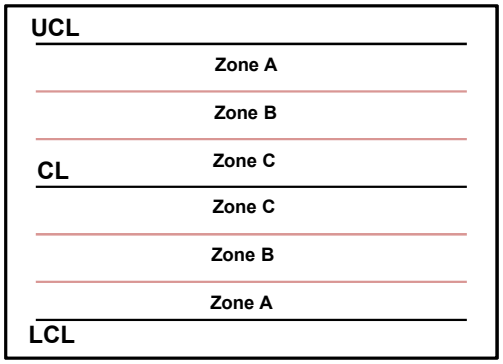
Samples 6, 8, 11, and 19 are out of control on the $\overline{X}$-chart and sample 9 is out of control on the *R* –chart.

39

Process Control Charts

Interpretation of $\overline{X}$ - and R – Charts:

When interpreting $\overline{X}$ - and R control Charts, each of the regions between the CL - UCL and CL - LCL are divided into three equal zones, corresponding to 1σ and 2σ intervals as shown below:



40

Process Control Charts

Rules:

1. **Whenever a single point falls outside the 3-sigma control limits, a lack of control is indicated. Since the probability of this happening is rather small, it is very likely not due to chance.**

2. **Whenever at least 2 out of 3 successive values fall on the same side of the centreline and more than 2-sigma units away from the centreline (in Zone A or beyond), a lack of control is indicated.**

3. **Whenever at least 4 out of 5 successive values fall on the same side of the centreline and more than one-sigma unit away from the centreline (in Zones A or B or beyond), a lack of control is indicated.**

4. **Whenever at least 8 successive values fall on the same side of the centreline, a lack of control is indicated.**

41

Thank You

42