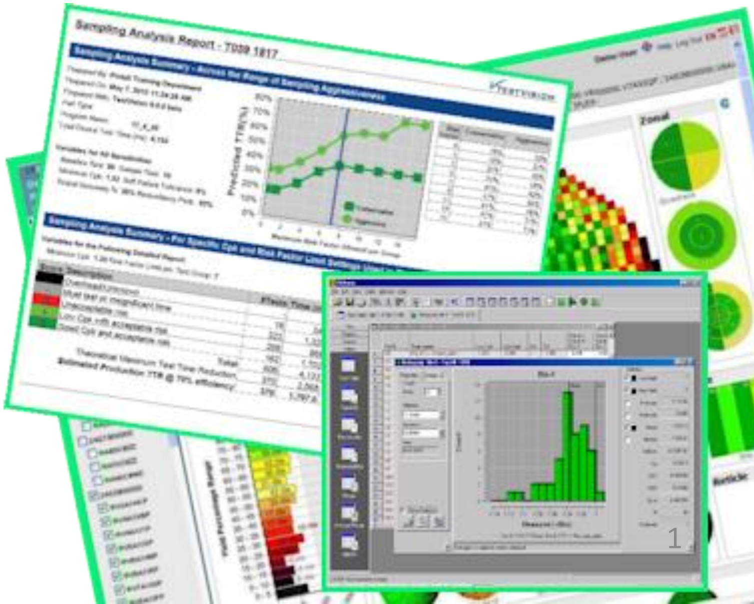
Assessment and Analysis of Test Data

Presented by SARF

Presenter:
Ron Berkers



Module A
Presentation of Data Sets

Numbers

Natural numbers (N):

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 etc.

Integers (Z):

No fractional or decimal component

21, 4, and −2048 are integers
9.75, 5½, and √2 are not integers

consists of the natural numbers 0, 1, 2 ,3 , ...
and
the negatives of the non-zero natural numbers −1, −2, −3, ...

Numbers

Rational Numbers (Q):

can be written as a simple fraction (i.e. as a ratio)

$$\frac{p}{q}$$

Since *q* may be equal to 1, every integer is a rational number

Number	As a fraction	Rational?
5	5/1	yes
1.75	7/4	yes
0.001	1/1000	yes
0.111...	1/9	yes
√2 (square root of 2)	?	No!

Numbers

Irrational Numbers :

cannot be written as a simple fraction (i.e. as a ratio)

$$1.5 = \left.\frac{3}{2}\right\} \text{Ratio}$$

Rational

$$\pi = 3.14159\ldots\ldots = \left.\frac{?}{?}\right\} \text{No Ratio}$$

Irrational

Fundamental Operations

Summation Sign (Σ = sigma):

End at this value

$$\sum_{n=1}^{4} n = 1 + 2 + 3 + 4 = 10$$

Start at this value

What to sum

Examples:

$$\sum_{n=1}^{4}(2n+1) = 3+5+7+9 = 24$$

$$\sum_{i=1}^{3} i(i+1) = 1\cdot 2 + 2\cdot 3 + 3\cdot 4 = 20$$

$$\sum_{i=3}^{5} \frac{i}{i+1} = \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$$

Fundamental Operations

Average Least Dimension (ALD):

❖Measure at least 200 aggregate particles
❖Take the average of the 200 particles

$$ALD = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{200} x_1 + x_2 + x_3 + \ldots\ldots + x_{200}}{200}$$

Where:

x_i = the i-th ALD measurement (i.e. i = 1 to 200)

And

n = the number of chips measured (i.e. n = 200)

Fundamental Operations

Logarithms

A logarithm answers the question:
How many of *one number* do we multiply to get *another number?*

Example:

How many 2s do we multiply to get 8?
Answer: 2 × 2 × 2 = 8, so we needed to multiply 3 of the 2s to get 8
So the logarithm is 3

$\boxed{\log_2(8) = 3}$ **because** $\boxed{2^3 = 8}$

Look at it this way:

$$\boxed{\begin{array}{c} 2^3 = 8 \\ \log_2(8) = 3 \end{array}}$$

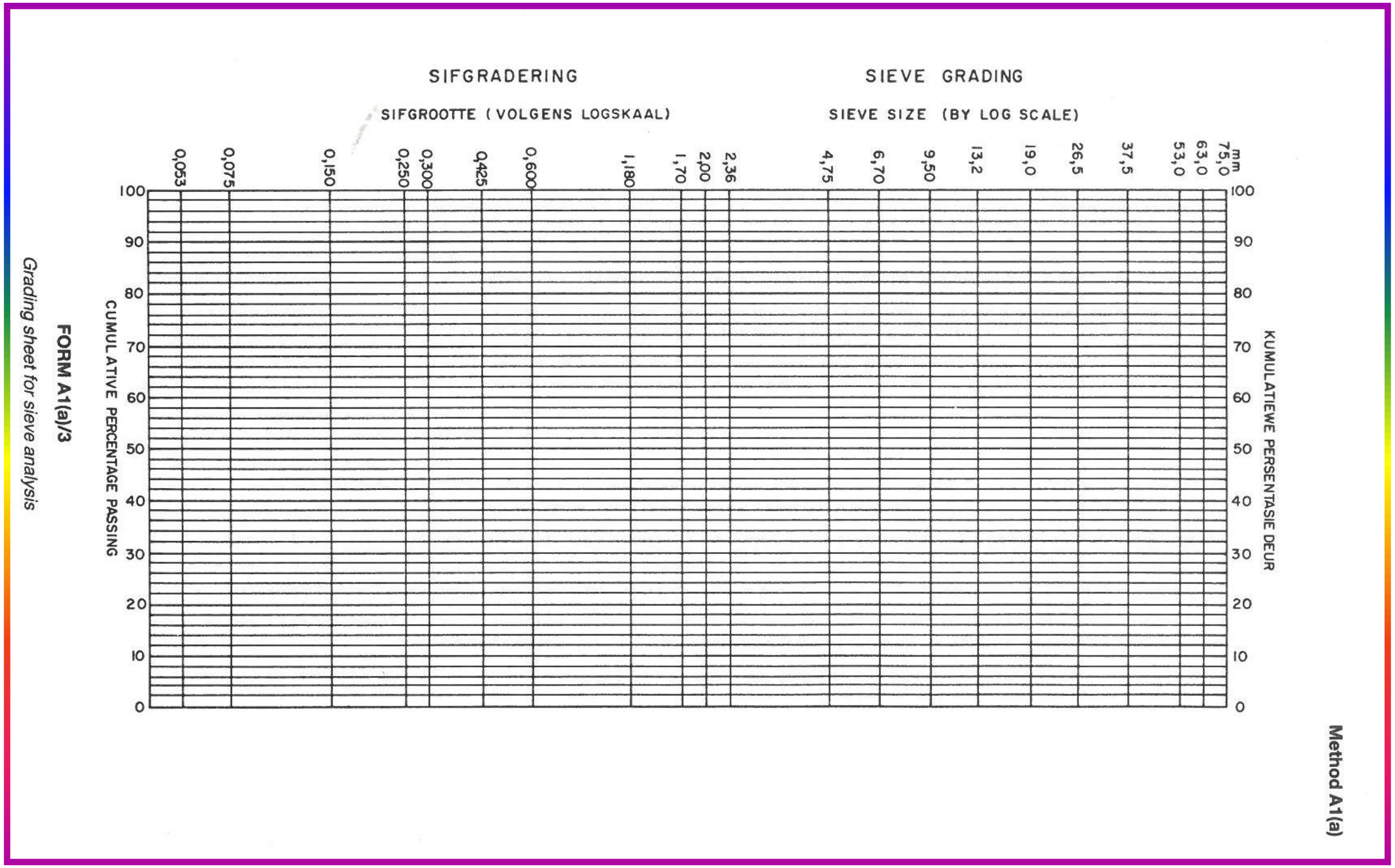
or

$$\boxed{\begin{array}{c} a^x = y \\ \log_a(y) = x \end{array}}$$

Fundamental Operations

Logs are also used in certain soil analysis procedures, such as:

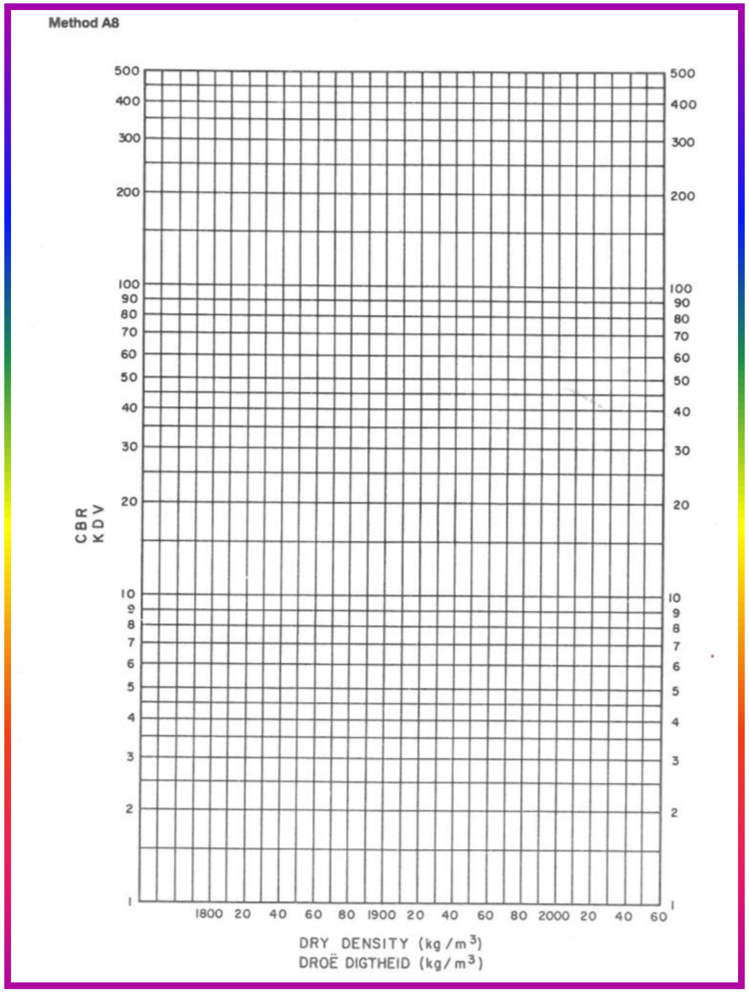
- **Representing cumulative particle size distribution (gradings) graphically, the scale indicating the sieve sizes is logarithmic (TMH 1: Method A1(a))**



Fundamental Operations

Logs are also used in certain soil analysis procedures, such as:

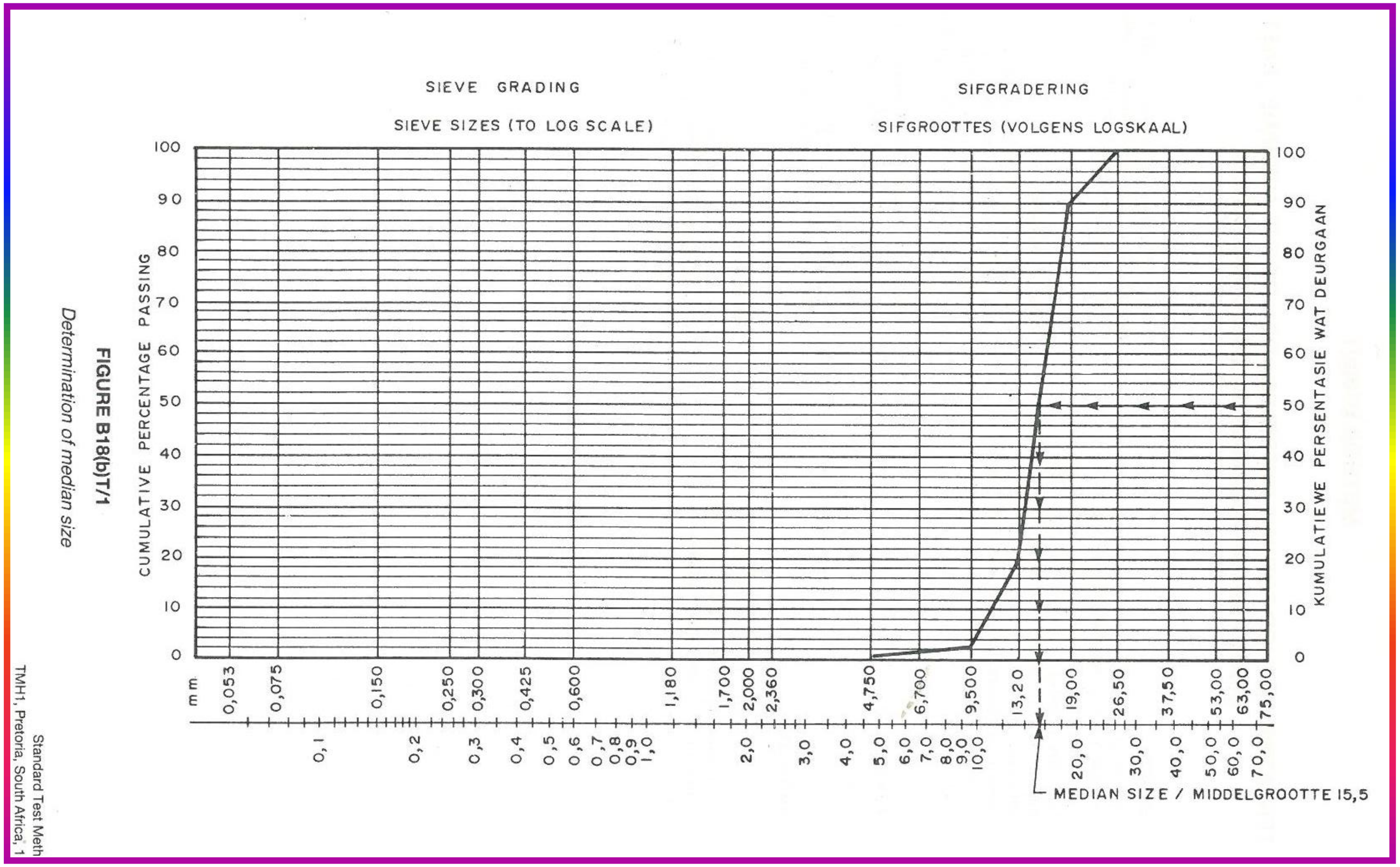
- **CBR versus density (compaction), the CBR scale is logarithmic (TMH 1: Method A8)**



Fundamental Operations

Logs are also used in certain soil analysis procedures, such as:

- **To determine the median size of an aggregate grading, the function contains logarithmic variables (TMH1: Method B18(b)T)**



Fundamental Operations

Random Numbers

Random numbers are numbers that occur in a sequence such that two conditions are met:

(1) the values are uniformly distributed over a defined interval or set, e.g.

Make use of set of real numbers between two prescribed numbers:

$$\{x: a \leq x \leq b\}$$

where:

$$a = 1; b = 9$$

Random Numbers:

$$\{3,7,5,1,9,3,2,8,7,5\}$$

(2) it is impossible to predict future values based on past or present ones

sample electromagnetic noise generated by the chaotic movement of electrons

Fundamental Operations

Random Numbers

- algorithms have been devised that supposedly generate random numbers.
- The problem with these methods is that they violate condition (2) in the definition of randomness.
- The existence of any number-generation algorithm produces future values based on past and/or current ones.
- Digits or numbers generated in this manner are called pseudorandom

Fundamental Operations

Random Numbers

Frequent use will be made of sets of real numbers between two prescribed real numbers. Such sets are called bounded intervals

the prescribed real numbers are called the boundary points

[and] imply the endpoint is included

(and) imply the endpoint is not included

Open Interval: (a , b) boundary points not included

where:

$$\{x\!: \; a < x < b\}$$

Fundamental Operations

Random Numbers

Closed Interval: [a , b] boundary points are included

where:

$$\{x:\ a \le x \le b\}$$

Right - Closed Interval: (a , b] a is not included, b is included

where:

$$\{x:\ a < x \le b\}$$

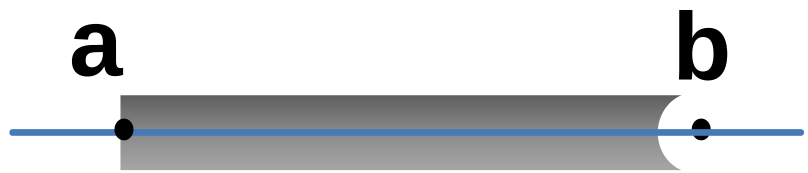
Fundamental Operations

Random Numbers

Left - Closed Interval: [a , b) **a is included, b is not included**

where:

$$\{x:\ a \leq\ x\ <\ b\}$$



unbound

Set of Real Numbers R: (∞^- , ∞^+)

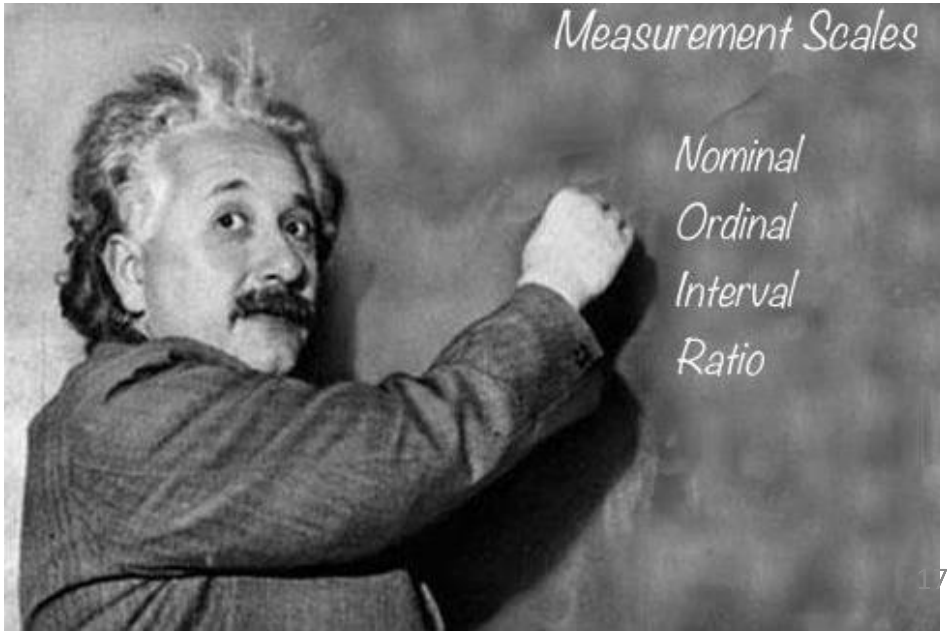
infinite negative real numbers

infinite positive real numbers

Fundamental Operations

Measurement Scales

- Measurement is the assignment of numbers to objects or events (variables) in a systematic fashion.
- Measurement scales are used to categorize and/or quantify these objects or events.
- Different measurement scales allow for different levels of exactness, depending upon the characteristics of the variables being measured.
- The scale is chosen depending on the information that the data is intending to represent.
- Four levels of measurement:



Fundamental Operations

Measurement Scales

Nominal Scale:

❖Used for labelling variables, without any quantitative value
❖"Nominal" scales could simply be called "labels."
❖Data is categorized and can't be arranged in an order from low to high

Examples:

Distress of a pavement is identified as:
"1-fracture" (cracking) or
"2-Deformation" (rutting)

Fundamental Operations

Measurement Scales

Ordinal Scale:

- **More precise than nominal scales**
- **Are basically sets of rankings**
- **Do not know the size of the differences between any data points, just that one is greater than the other**
- **Typical relations between classes are: “higher”, “stronger”, “more difficult”, “more disturbed”**

Example (1):

Pavement standard:

(1) materials complying with base course standard

(2) materials complying with sub base standard

base course standard higher (>) than sub base standard

Fundamental Operations

Measurement Scales

Example (2): Mohs scale of hardness of minerals

Mohs value	Mineral
1	Talc
2	Gypsum
3	Calcite
4	Fluorite
5	Apatite
6	Orthoclase
7	Quartz
8	Topaz
9	Corundum
10	Diamond

Do not say : Apatite = 5 (5 is not an operational number, just point on scale, could just as easily have been (e))
Rather say: Hardness of Apatite is 5 on the Mohs scale

Measurement Scales

Interval Scale:

- ❑keeps the same rank characteristic as ordinal scales
- ❑interval scales also show the differences between each data point, i.e.the distances between any two numbers on the scale are of known size
- ❑allows for the *degree of difference* between items, but not the ratio between them

Example:

Thermometer readings on a Celsius scale.

The difference between 98.6 and 99.6 is the same as the difference between 101.8 and 102.8 – i.e. 1 degree.

The value of zero doesn't mean "the absence of heat."

Fundamental Operations

Measurement Scales

Ratio Scale:

- The most precise and powerful of scales
- Ratio scales have all the components of an interval scale but here, the zero point is meaningful and means the absence of whatever it is you're measuring
- Thus, you cannot have a negative data point using a ratio scale

Examples:

A score of 20 is 20 times greater than 1 (20:1) and 10 times greater than 2 (20:2) and twice as great as 10 (20:10)

Measuring something with a ruler would give you a measure in a ratio scale. Zero literally means "no length" (i.e., that it doesn't exist)

The cost of a cup of coffee, 2 cups are twice as expensive as 1 cup

Fundamental Operations

ISO / SI System

- The International System of Units is called the SI System(abbreviated *SI* from *Systeme Internationale* (the French version of the name)
- It is a scientific method of expressing the magnitudes or quantities of important natural phenomena
- It is the modern form of the metric system
- It comprises a coherent system of units of measurement built around seven base units (Table 1):

Measurement	Unit	Symbol
Mass (not weight)	kilogram	kg
Length	meter	m
Temperature	kelvin	K
Time	second	s
Pure substance amount	mole	mol
Electric current	ampere	A
Light brightness (wavelength)	candela	cd

Fundamental Operations

ISO / SI System

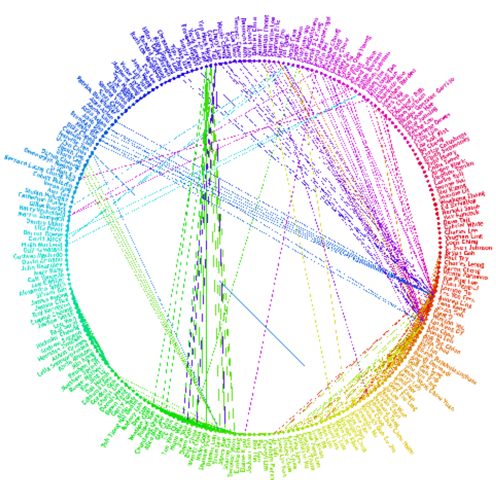
- 22 named and an indeterminate number of unnamed coherent derived units (Table 2)

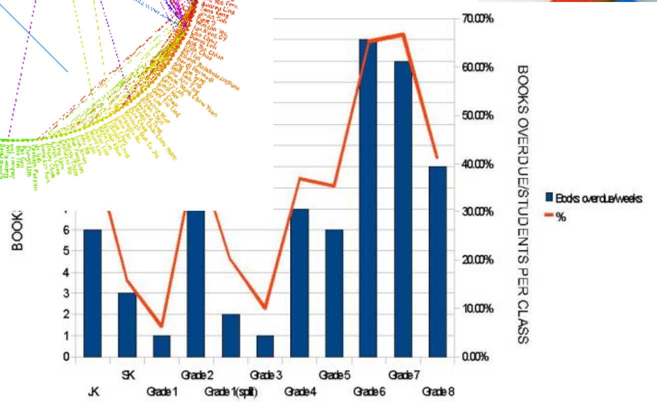
Name	Symbol	Quantity	Expression in terms of other units	Expression in terms of SI Base units
becquerel	Bq	radioactivity (decays per units time)	1/s	s^{-1}
coulomb	C	electric charge or quantity of electricity	s•A	s•A
degree Celsius	°C	temperature relative to 273.15 K	K	K
farad	F	electric capacitance	C/V	$kg^{-1} \cdot m^{-2} \cdot s^{4} \cdot A^{2}$
gray	Gy	absorbed dose of ionizing radiation	J/kg	$m^{2} \cdot s^{-}$
henry	H	inductance	V•s/A = Wb/A	$kg \cdot m^{2} \cdot s^{-2} \cdot A^{-2}$
hertz	Hz	frequency	1/s	s^{-1}
joule	J	energy, work, heat	N•m = C•V = W•s	$kg \cdot m^{2} \cdot s^{-2}$
katal	kat	catalytic activity	mol/s	$s^{-1} \cdot mol$
lumen	lm	luminous flux	cd•sr	cd
lux	lx	illuminance	lm/m^2	$m^{-2} \cdot cd$
newton	N	force, weight	$kg \cdot m/s^2$	$kg \cdot m \cdot s^{-2}$
ohm	Ω	electric resistance, impedance, reactance	V/A	$kg \cdot m^{2} \cdot s^{-3} \cdot A^{-2}$
pascal	Pa	pressure, stress	N/m^2	$kg \cdot m^{-1} \cdot s^{-2}$
radian	rad	angle	m/m	dimensionless
siemens	S	electrical conductance	1/Ω = A/V	$kg^{-1} \cdot m^{-2} \cdot s^{3} \cdot A^{2}$
sievert	Sv	equivalent dose of ionizing radiation	J/kg	$m^{2} \cdot s^{-2}$
steradian	sr	solid angle	m^2/m^2	dimensionless
tesla	T	magnetic field strength, magnetic flux density	$V \cdot s/m^2$ = Wb/m^2 = N/(A•m)	$kg \cdot s^{-2} \cdot A^{-1}$
volt	V	voltage, electrical potential difference, electromotive force	W/A = J/C	$kg \cdot m^{2} \cdot s^{-3} \cdot A^{-1}$
watt	W	power, radiant flux	J/s = V•A	$kg \cdot m^{2} \cdot s^{-3}$
weber	Wb	magnetic flux	J/A	$kg \cdot m^{2} \cdot s^{-2} \cdot A^{-1}$

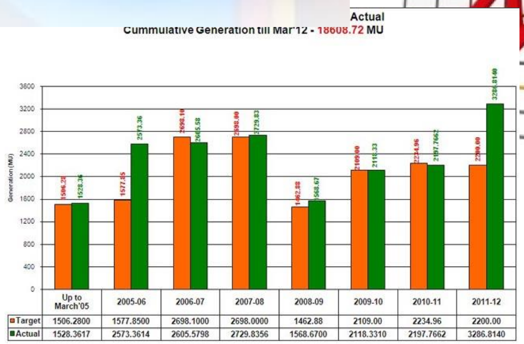
Fundamental Operations

Graphical Representation

- A method of visually creating and manipulating data and statistical results
- Used in order to gain better insight and understanding of a problem that is being studied
- pictures can convey an overall message much better than a list of numbers





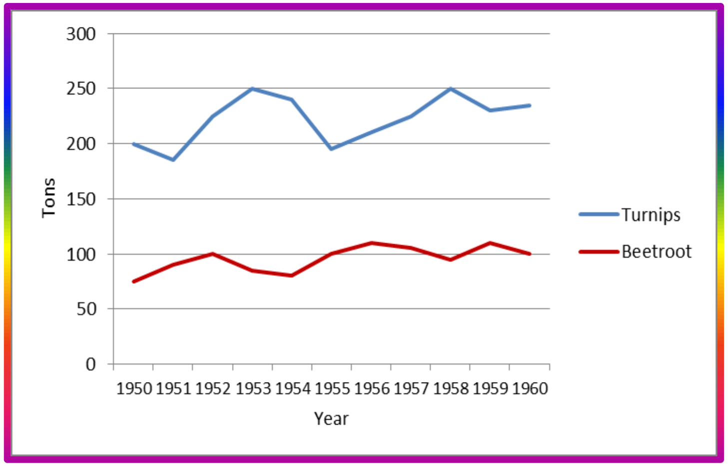


Fundamental Operations

Graphical Representation

Line Chart:

✓A line chart plots continuous data as points and then joins them with a line
✓provide a simple way to visually present a sequence of many values
✓Used to display long data rows
✓Can interpolate between data points
✓Can extrapolate beyond known data values (forecast)
✓Used to compare different graphs
✓to find and compare trends (changes over time)
✓if the X axis requires an interval scale
✓to display interactions over two levels on the X-axis

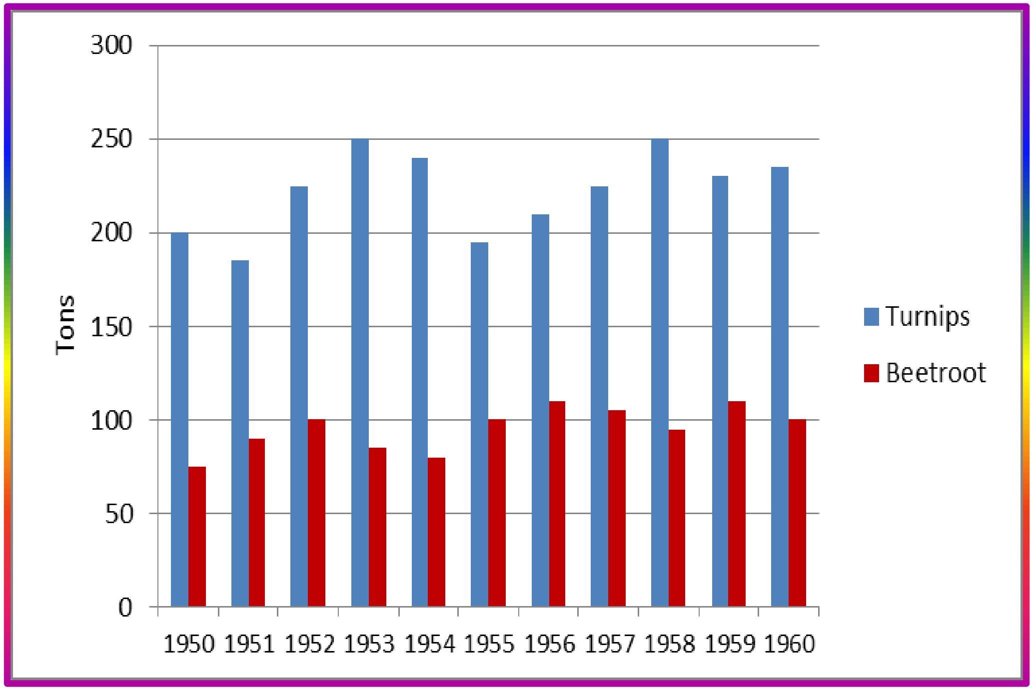


Fundamental Operations

Graphical Representation

Bar Chart:

- A bar chart displays discrete data in separate columns
- A double bar chart can be used to compare two data sets
- Bar charts are best to show amounts
- Can be plotted vertically or horizontally



Fundamental Operations

Graphical Representation

Histogram:

- a histogram is a graphical representation showing a visual impression of the distribution of data
- It is similar to a Bar Chart, but a histogram groups numbers into ranges
- There should be no gaps between the bars in the histogram

Example:

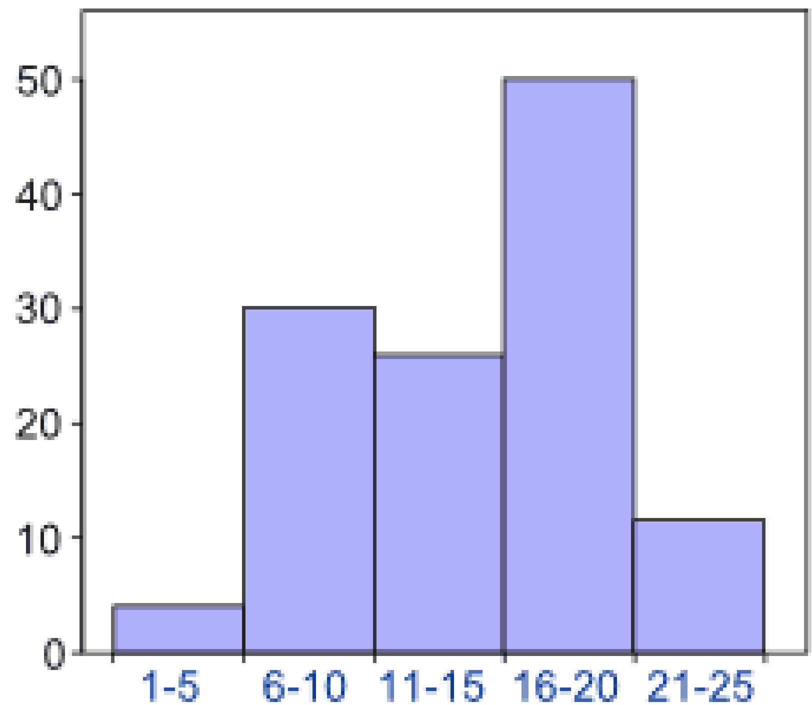
You do a survey of your roads data base and want to find out how many roads of a certain length are in a certain district
You decide to divide the roads into lengths of 5 kilometres
The following results were obtained:

1 to 5 kilometres: 3 roads
6 to 10 kilometers: 30 roads
11 to 15 kilometers: 27 roads
16 to 20 kilometers: 50 roads
21 to 25 kilometers: 10 roads

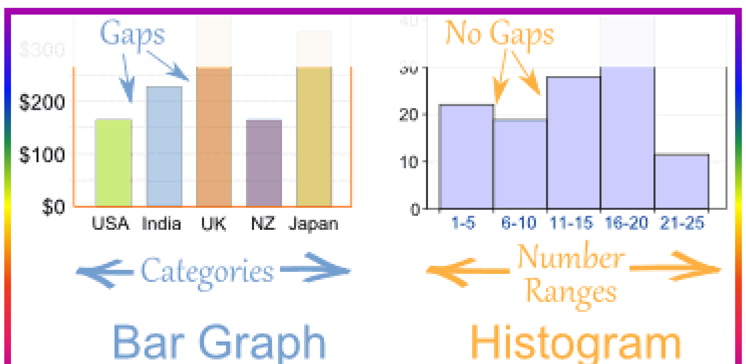
Fundamental Operations

Graphical Representation

Histogram:



There are 30 roads between 6 and 10 kilometres long



Fundamental Operations

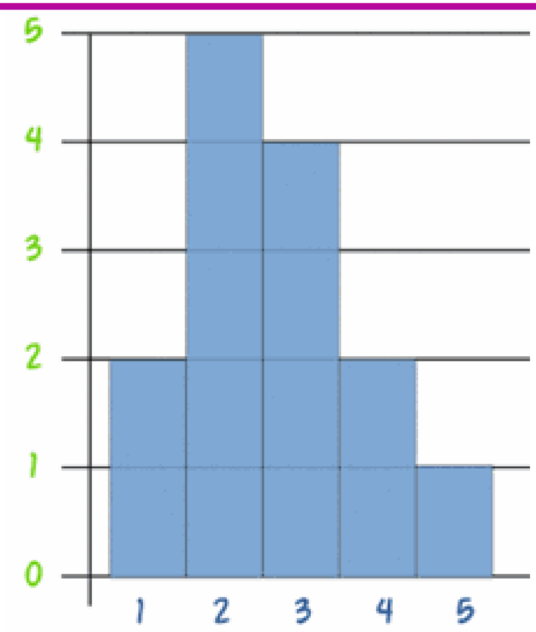
Graphical Representation

Frequency histograms (also frequency distribution):

A Frequency Histogram is a special histogram that uses vertical columns to show frequencies (how many times each score occurs)

Example:

You get the following scores: 1,1,2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5



The frequency of number 2 is 5
i.e. The number 2 occurs 5 times

Fundamental Operations

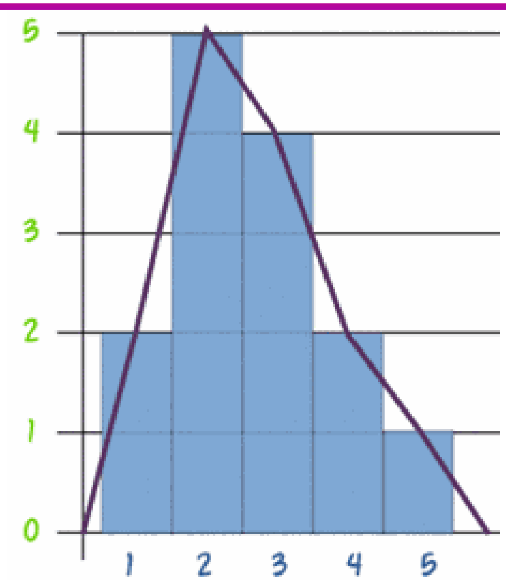
Graphical Representation

Frequency polygons:

The frequency of each class is indicated by points or dots drawn at the midpoints of each class interval. Those points are then connected by straight lines

Example:

Consider the same scores from the previous example.



A graph made by joining the middle-top points of the columns of a frequency histogram

Fundamental Operations

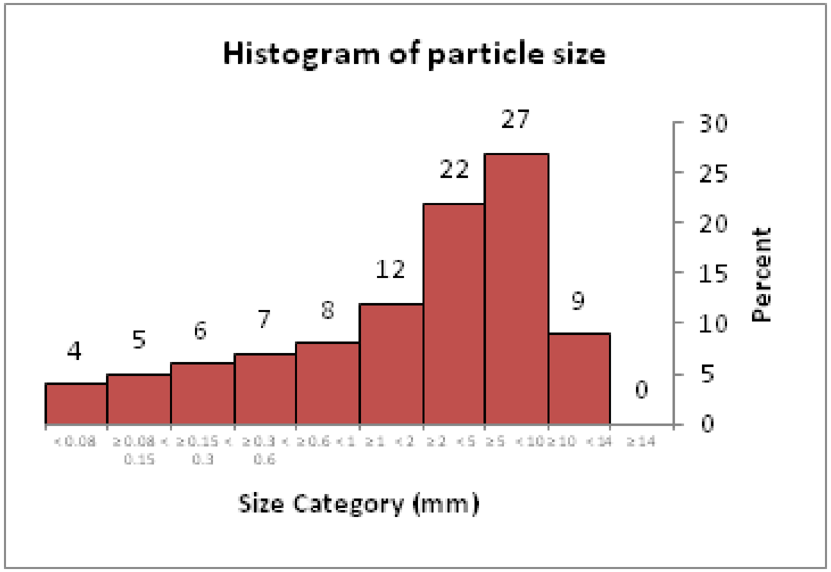
Graphical Representation

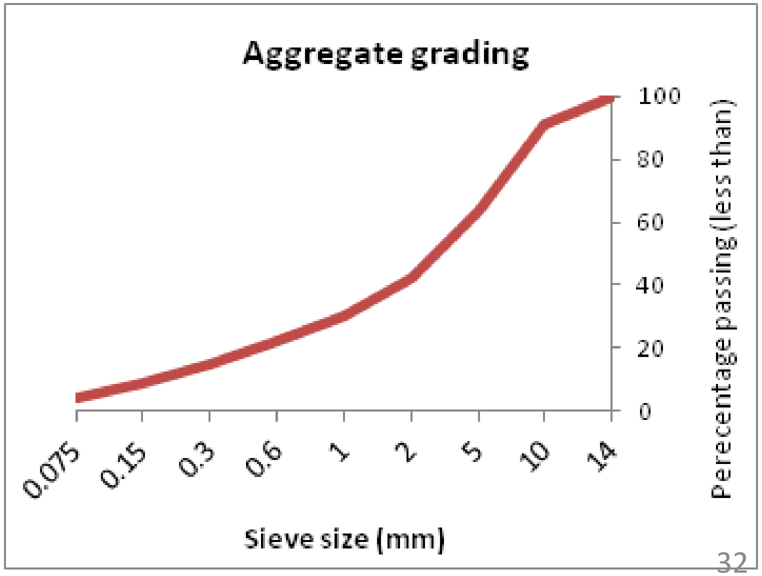
Cumulative frequency polygon:

a way of presenting data so that frequencies are accumulated as the value of data increases

Example:

The grading curve





Fundamental Operations

Graphical Representation

Logarithmic Scale:

The logarithmic scale on a graph is used to accommodate large numbers and very small numbers on the graph in such a way that all the numbers can easily be distinguished from each other on one or both axes of the graph.

Example:

The grading curve:

- **The y-axis is normally indicated as cumulative percentage passing through the various sieves.**
- **The X-axis is indicated as a 3 cycle log scale, where the x-axis is divided into 3 regions between 0 an 1, 1 and 10 and 10 and 100 (0, 10^0, 10^1 and 10^2 respectively). (5 cycle if hydrometer test is done on the soil).**
- **If this would not have been implemented, the 0,075mm to 0,600mm sieve would be unreadable on the graph compared to the sieves above 4,75mm sieves (would be seen as a point).**

Fundamental Operations

Graphical Representation

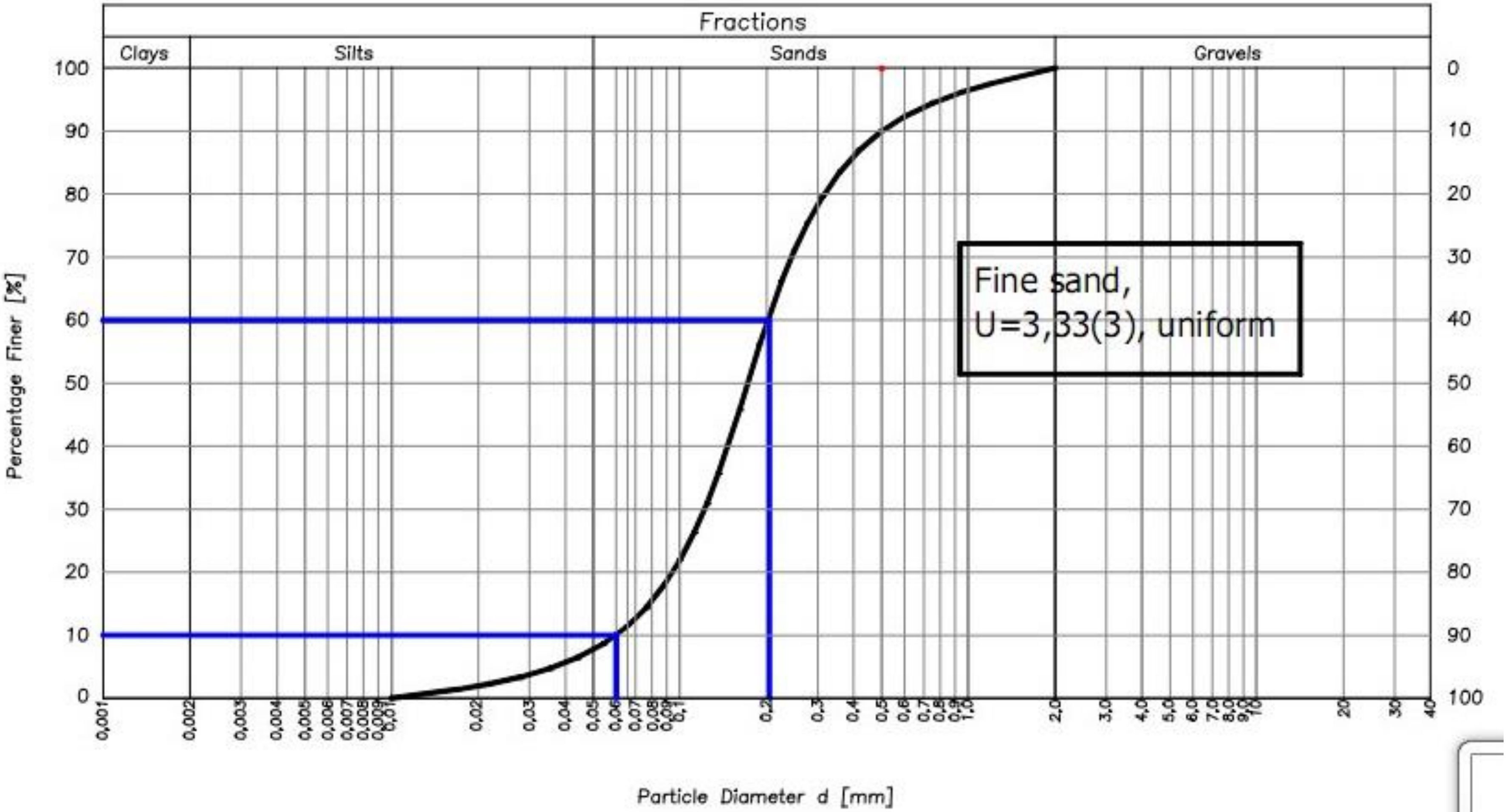


Figure 1. Particle-size distribution curve.

End of Session A

Time to rest

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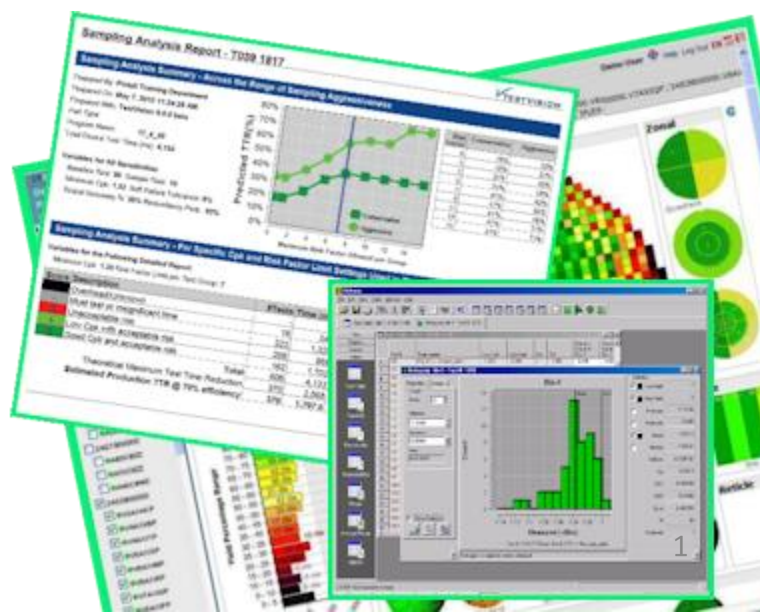
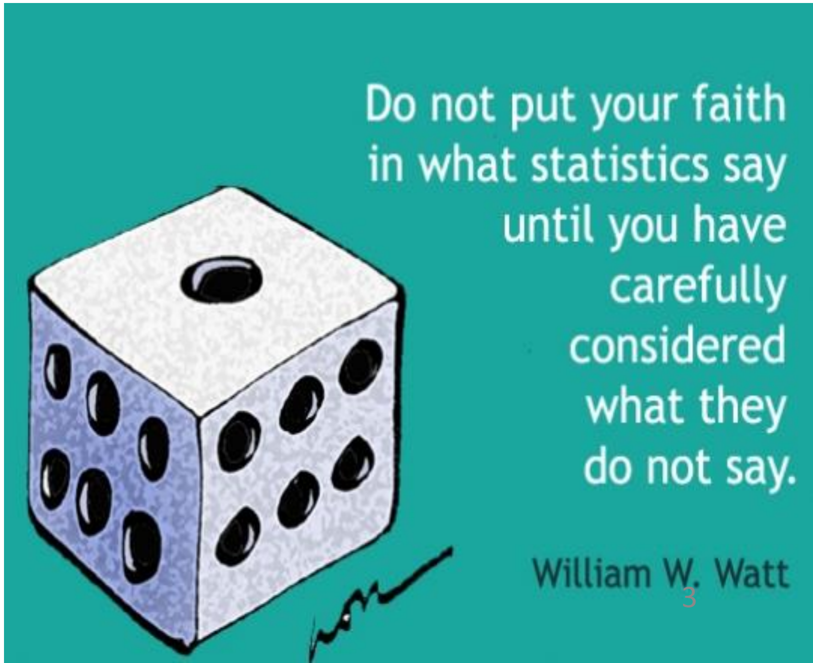


Module B
Statistical Methods

Statistics:

- ➢Collection of data
- ➢Organising of data
- ➢Summarising of data
- ➢Analysing of data
- ➢Interpretation of data
- ➢Presenting of data



Do not examine the whole group - called the population

Examine only part of the group – called the sample

Example:

You will not test the whole soil layer, you will take a sample consisting of 2 bags of 50 kg each. (Representative sample)

Central tendency

Average

- a measure of the "middle" or "typical" value of a data set
- thus a measure of central tendency

- Mean (or arithmetic mean)
- Median
- Mode

Central tendency

Mean:

- The mean is the average of the numbers.
- add up all the numbers, then divide by how many numbers there are.

$$\bar{X} = \frac{\sum_{j=1}^{n} x_j}{n}$$

Example: What is the Mean of these numbers?

6, 11, 7

- Add the numbers: 6 + 11 + 7 = 24
- Divide by *how many* numbers (there are 3 numbers): 24 / 3 = 8

The Mean is 8

- 6, 11 and 7 added together is the same as 3 lots of 8

Central tendency

Median:

- Defined as the "middle" value of the set when the values are arranged in order.

Example: Consider the following numbers:

{3, 13, 7, 5, 21, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29}

- If we put those numbers in order we have:

{3, 5, 7, 12, 13, 14, 21, 23, 23, 23, 23, 29, 39, 40, 56}

- There are fifteen numbers. Our middle number will be the eighth number:

{3, 5, 7, 12, 13, 14, 21, **23**, 23, 23, 23, 29, 39, 40, 56}

- The median value of this set of numbers is 23.

(Note that it didn't matter if we had some numbers the same in the list)

Central tendency

Example: **Two Numbers in the Middle**

What if there are an even amount of numbers?

In that case we need to find the middle pair of numbers, and then find the value that would be half way between them.
This is easily done by adding the middle pair together and dividing by two.

- **Consider the following numbers:**

{3, 13, 7, 5, 21, 23, 23, 40, 23, 14, 12, 56, 23, 29}

- **If we put those numbers in order we have:**

{3, 5, 7, 12, 13, 14, 21, 23, 23, 23, 23, 29, 40, 56}

Central tendency

- There are now fourteen numbers and so we don't have just one middle number, we have a pair of middle numbers:

{3, 5, 7, 12, 13, 14, **21**, **23**, 23, 23, 23, 29, 40, 56}

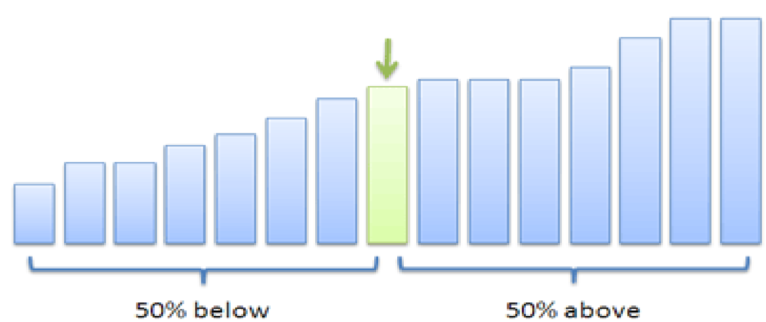
- In this example the middle numbers are 21 and 23.

- To find the value half-way between them, add them together and divide by 2:

$$21 + 23 = 44$$
$$44 \div 2 = 22$$

- And, so, the Median in this example is 22.

Median



Central tendency

Mode:

- The Mode is the value that occurs most often.

Example:

- Consider the following numbers:

{13,13,2,13,2,2,13,2,13,2,13}

- Group the numbers so we can count them:

{2, 2, 2, 2, 2, 13, 13, 13, 13, 13, 13}

- "13" occurs 6 times, “2" occurs only 5 times
- The mode is 13.

Central tendency

Example:

- Consider the following numbers:

{3, 6, 4, 7, 6, 4, 5}

- Group the numbers so we can count them:

{3, 4, 4, 5, 6, 6, 7}

- “4" and “6” both occur 2 times
- Both 4 and 6 are modes.
- This is called “Bimodal”
- 3 or more modes is called “Multimodal”

Central tendency

Quantile:

- We often want to summarize a set of numbers in a few numbers, for ease of reporting or comparison. The most direct method is to use quantiles. The quantiles are values which divide the set of numbers such that there is a given proportion of observations below the quantile.

- For example, the median is a quantile. The median is the central value of a set of numbers, such that half the points are less than or equal to it and half are greater than or equal to it.

Central tendency

Quartile:

- **Quartiles divide a set of numbers (distribution) into four equal parts.**

Example:

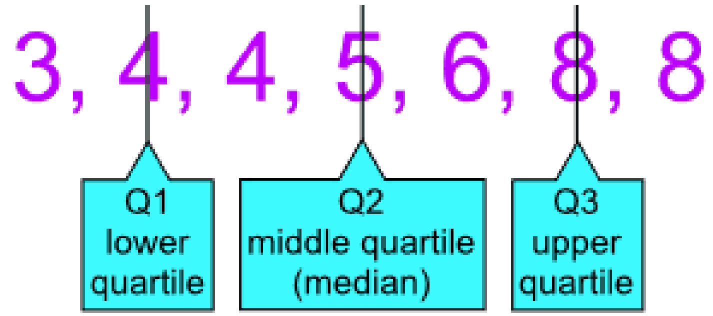
- **Consider the following numbers:**

$$\{5, 8, 4, 4, 6, 3, 8\}$$

- **Put them in order:**

$$\{3, 4, 4, 5, 6, 8, 8\}$$

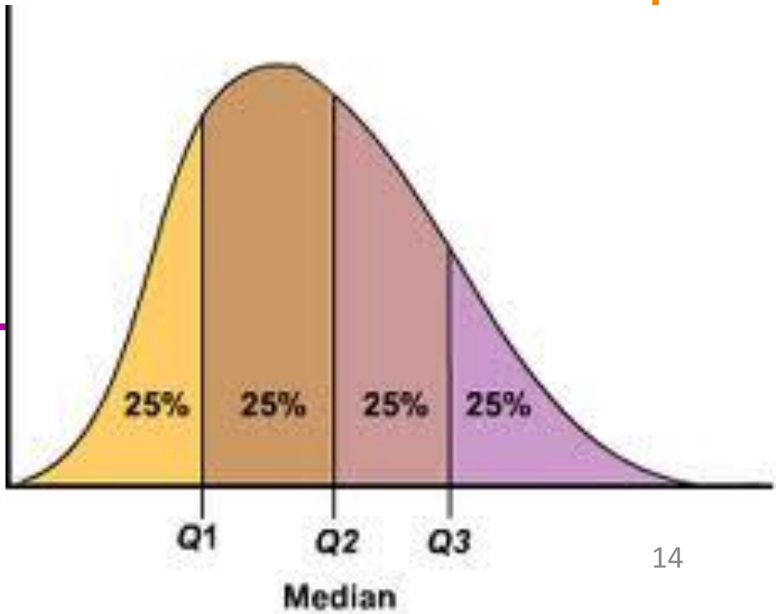
▪Cut the list into quarters:



▪Q1 = 4

▪Q2 = 5 (which is also the median)

▪Q3 = 8



Measures of Dispersion

Dispersion:

- **Measures of central tendency are used to estimate "normal" values of a dataset**
- **Measures of dispersion are important for describing the spread of the data, or its variation around a central value**
- **Two distinct samples may have the same mean or median, but completely different levels of variability, or vice versa**

Example:

10 homes with a value of R500000 have the same mean as 2 homes with a value of R750000 and 8 homes with a value of R125000.
The mean will not show the variation between the prices of the houses.

V/S

Measures of Dispersion

Range:

- The simplest measure of dispersion.
- The range is calculated by taking the difference between the maximum and minimum values in the data set.
- The range only provides information about the maximum and minimum values and does not say anything about the values in between.

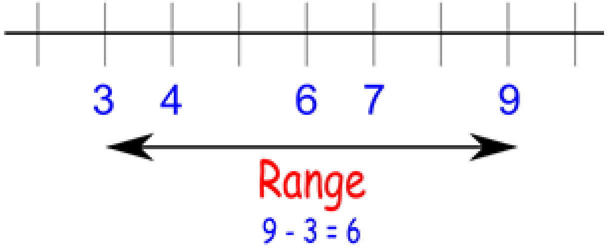
Example:

- Consider the following numbers:

{4, 6, 9, 3, 7}

- The lowest value is 3, and the highest is 9.

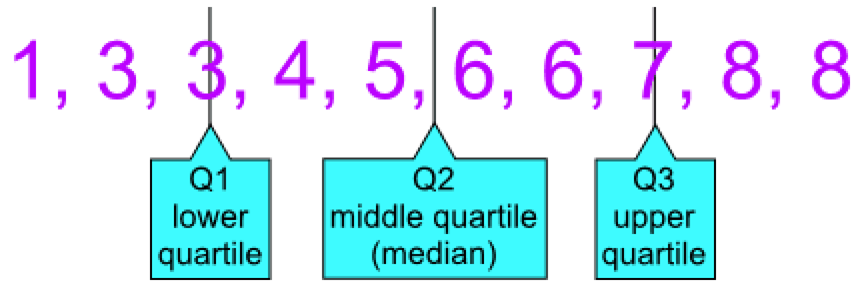
- The range is: 9 – 3 = 6



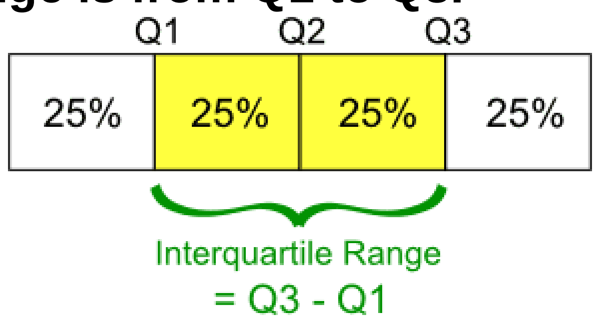
Measures of Dispersion

Inter – Quartile Range:

- A good indicator of the spread in the centre region of the data
- More resistant to extreme values than the range



- The inter-quartile range is from Q1 to Q3.



- Quartile 1 (Q1) = 3
  Quartile 2 (Q2) = 5.5
  Quartile 3 (Q3) = 7

- The Inter-quartile Range is:

  Q3 - Q1 = 7 - 3 = 4

Measures of Dispersion

Mean Deviation:

- Uses each data value
- It is the mean of the distances between each value and the mean
- It gives us an idea of how spread out from the centre the set of values is

$$\overline{\text{Dev}} = \frac{\sum_{j=1}^{N} |x - \bar{x}|}{N}$$

Example:

- Consider the following numbers:

{9,2,7,5,4,6,5,2}

- The mean of these values is:

(9+2+7+5+4+6+5+2)/8= 5

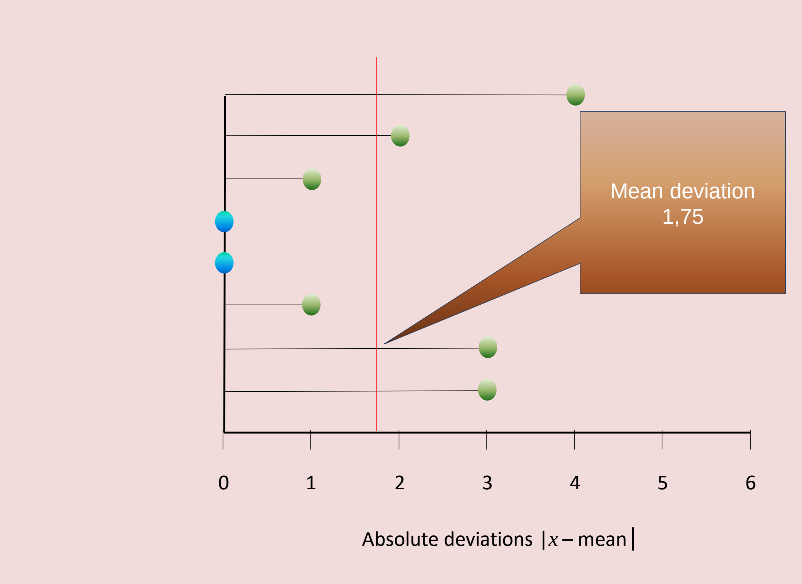
- Each value can now be expressed as a deviation from the mean by subtracting the mean from it:

(9-5, 2-5, 7-5, 5-5, 4-5, 6-5, 5-5, 2-5)

= (4, -3, 2, 0, -1, 1, 0, -3)

▪Negative signs are ignored (i.e. the absolute deviations are considered), hence:

$$\{4 \quad 3 \quad 2 \quad 0 \quad 1 \quad 1 \quad 0 \quad 3\}$$



▪The set of values are the original data expressed as absolute deviations from the mean. The mean of these absolute deviations can be calculated as:

$$(4+3+2+0+1+1+0+3)/8 = 1{,}75$$

Measures of Dispersion

Variance:

- **The average of the squared differences from the Mean.**
- **Not often used.**

$$\sigma^2 = \frac{\sum_{j=1}^{N}(x - \bar{x})^2}{N}$$

Example:

- **Consider again the following numbers:**

{9,2,7,5,4,6,5,2}

- **The variance for this set is:**

$$\sigma^2 = (4^2+(-3)^2+2^2+0^2+(-1)^2+1^2+0^2+(-3)^2)/8 = 5$$

Measures of Dispersion

Standard Deviation:

▪Standard Deviation is defined as the square root of the variance.

$$\sigma = \sqrt{\frac{\sum_{j=1}^{N}(x - \bar{x})^2}{N}}$$

$$s = \sqrt{\frac{\sum_{j=1}^{N}(x - \bar{x})^2}{N-1}}$$

Use σ for population and s for a sample

Measures of Dispersion

Example:

- **Consider again the following numbers:**

  {9,2,7,5,4,6,5,2}

- **The standard deviation for this set is:**

$$\sigma = \sqrt{(4^2+(-3)^2+2^2+0^2+(-1)^2+1^2+0^2+(-3)^2)/8} = 2,236$$

- **Consider a sample of this set:**

  {9,2,7)

- **The standard deviation of the sample of this set is:**

$$s = \sqrt{(4^2+(-3)^2+2^2/(3-1)} = 3,81$$

Measures of Dispersion

Example:

Sampling of concrete

- **Concrete must be sampled from that being used for construction (according to SANS 5861-2)**
- **Each sample (sufficient for the slump test and three test cubes) must be taken from a different batch of concrete chosen on a random basis.**
- **At least one sample must be taken from each day's placing and from at least every 50 m³ of concrete of each grade being placed.**
- **When the batch size is less than 2 m³ , the first sample of each grade shall be taken after at least three batches of this grade have been mixed and discharged.**

Measures of Dispersion

Acceptance Criteria:

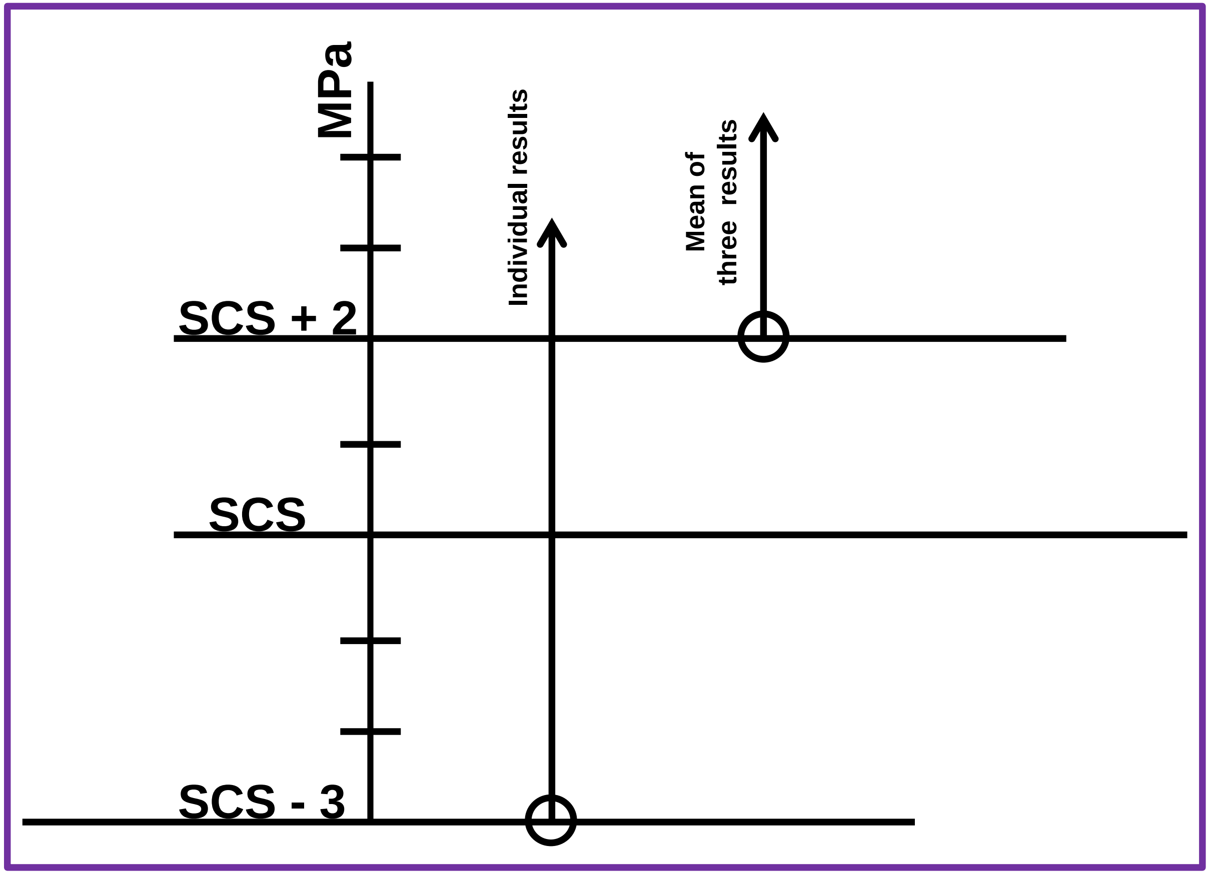
For a small numbers of result (number of values less than 30):

Strength test results shall meet <u>both</u> of the following acceptance criteria:

- **No individual result (average of three cube strength) shall be more than 3 MPa below the specified characteristic strength (SCS).**
- **The mean of any group of three consecutive and overlapping results shall exceed the specified characteristic strength by at least 2 MPa.**

Measures of Dispersion

Acceptance criteria for an individual result and mean of three consecutive results:



Measures of Dispersion

Acceptance Criteria:

For a large numbers of result (number of values more than 30):

- **The average of overlapping sets of 30 results for a specific grade of concrete shall exceed specified strength by 1,7 times the standard deviation.**

- **No individual result shall be more than 3 MPa below the specified characteristic strength.**

Measures of Dispersion

Process Control:

- **Specified Characteristic Strength (SCS) is defined as that strength below which no more than 5% of the valid results may fall.**

- **To ensure that no more than 5% of results are below SCS, a higher strength should be aimed at. This strength is called Target Average Strength (TAS).**

- **Roughly half of the valid results should be above TAS, and half below.**

- **To meet the above criteria, TAS should be 1,645 standard deviations above SCS.**

Measures of Dispersion

Process Control:

- **The standard deviation of the results will depend on the degree of control achieved during production. The better the control, the lower the standard deviation and, therefore, a lower TAS may be targeted.**

- **Below are given standard deviations and margins for different degrees of control as recommended by some experts as starting points for use at the beginning of a project:**

Degree of control	Standard deviation, Mpa	Margin, Mpa
Poor	7	11,5
Average	6	10,0
Good	5	8,5

Measures of Dispersion

Examples of strength result data: Specified Characteristic Strength is 35 MPa:

Test Number	Result, Mpa	Test Number	Result, Mpa
1	58	16	45
2	56	17	44
3	57	18	44
4	45	19	45
5	48	20	52
6	46	21	46
7	47	22	46
8	45	23	48
9	40	24	53
10	46	25	51
11	50	26	49
12	50	27	48
13	44	28	39
14	48	29	45
15	39	30	50
		Sum:	**1424**

Measures of Dispersion

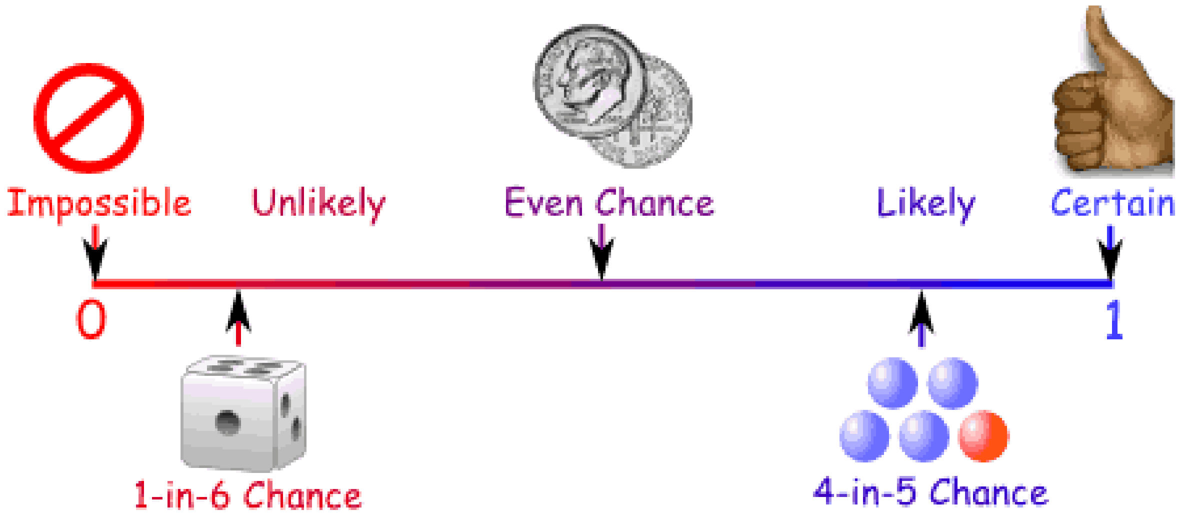
For the previous results:

- **Mean = $\bar{x}$ = 1424/30 = 47,5 MPa**
- **Standard Deviation = s = 4,7 MPa (all results – mean, sum of squared numbers/(30-1), square root of previous result)**
- **The value is smaller than the value of 6 taken for average control.**
- **The results therefore show a degree of control closer to "good" than to "average".**
- **The margin, to the nearest 0,5 MPa, should therefore be 1,7 x 4,7 = 8 MPa and not 10 MPa.**
- **TAS should be 43,0 MPa (minimum $\bar{x}$ = SCS + 8 = 35 + 8 = 43 MPa).**
- **According to this assessment, average strength is 47,5 – 43 = 4,5 MPa higher than required.**
- **Concrete to strong, reduce cement content (increase w:c), more economical**

Probability

Probability P(*A*):

- **Probability is the chance that something will happen - how likely it is that some event will happen.**
- **A probability is a way of assigning every event a value between zero and one, with the requirement that the event made up of all possible results is assigned a value of one**



Probability

Independent probability:

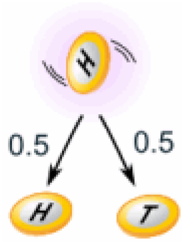
Calculated when the events are not affected by the previous events.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example:

What are the chances of the side of two coins being both heads?

Probability = ½ x ½ = ¼



Probability

Mutually exclusive events:

Mutually Exclusive means you can't get both events at the same time.

$$P(A \text{ or } B) = P(A) + P(B)$$

Example:

What are the chances of rolling a 1 or 2 on a six-sided dice?

$$\text{Probability} = P(1 \text{ or } 2) = P(1) + P(2)$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1}{3}$$

Probability

Not mutually exclusive events:

Not Mutually Exclusive means you can get both events at the same time.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example:

When drawing a single card at random from a regular deck of cards, what are the chances of getting a heart or a face card (J,Q,K), or one that is both?

Probability = P(H or F) – P(H and F)

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$= \frac{11}{26}$$

52 cards; 13 are hearts; 12 are face cards; 3 are hearts and face cards

Probability

Conditional Probability:

Conditional probability means that one event only happens after another has happened.

Probability = P(A) x P(*B*|*A*)

Example:

2 blue and 3 red marbles are in a bag. What are the chances of getting a blue marble first (event A) and second (event B)?

Probability of getting blue marble first (P(*A*): $\frac{2}{5}$

Blue marble first, chances of getting another blue marble P(B|A): $\frac{1}{4}$

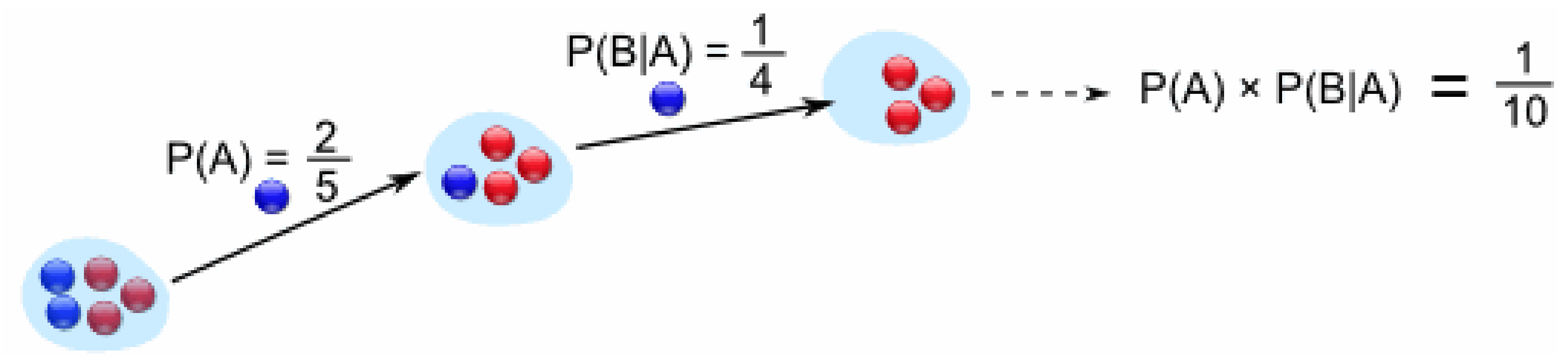
If we got a red marble first, chances of getting a blue marble: $\frac{2}{4}$

Probability

Conditional Probability = P(A) x P(*B*|*A*)

$$= \frac{2}{5} \times \frac{1}{4}$$

$$= \frac{1}{10}$$



Sampling

Random Sampling (Probability Sampling):

- Test points must be chosen by unpredictable means.
- Samples must be chosen so as to be representative of a population.
- A fundamental characteristic of random sampling is that each member of a population has an equal chance of being included in the sample.
- Acceptance sampling is used to determine if a production lot of material meets the governing specifications.

Sampling

Simple random Sampling:

- The simplest method of probability sampling.
- Within a particular study population everyone has an equal chance of inclusion in the sample.
- Considered ‘fair’ and therefore allows findings to be generalized to the whole population from which the sample was taken.
- Applicable when population is small, homogeneous & readily available.
- A lottery system or table of random numbers is used to determine which units are to be selected.

	1 2 3 4	5 6 7 8	9 10 11 12	13 14 15 16	17 18 19 20	21 22 23 24	25 26 27 28	29 30 31 32
1	8 0 9 4	2 5 2 5	8 2 4 7	1 3 4 7	7 4 3 3	3 6 2 0	1 8 9 7	2 1 3 4
2	3 5 6 3	2 1 9 8	8 2 1 1	9 0 4 5	2 6 1 8	2 7 5 1	2 6 2 7	1 0 9 5
3	1 3 3 0	6 3 3 1	3 7 5 3	9 6 9 3	8 7 3 8	6 6 1 5	1 5 3 8	8 5 4 3
4	3 5 6 5	0 0 1 6	2 2 4 3	6 4 3 2	4 7 9 6	6 0 9 5	5 2 8 3	1 6 2 0
5	7 8 5 0	5 9 2 6	5 5 8 8	7 3 1 1	2 1 9 2	4 5 4 5	3 5 3 0	5 5 8 9
6	4 4 9 0	5 4 1 7	9 7 2 7	6 1 5 3	5 9 0 1	4 8 7 8	9 9 8 0	9 8 7 7
7	6 6 4 6	9 1 0 4	9 3 1 8	8 8 1 9	7 5 3 7	2 7 8 5	9 3 7 3	2 4 4 5
8	3 6 2 6	5 9 9 5	1 2 1 5	9 7 5 3	9 2 2 3	5 6 5 8	2 9 4 4	2 8 9 9
9	4 6 6 5	4 8 2 0	7 5 5 4	0 6 1 2	9 6 8 3	4 2 5 1	9 1 3 8	1 7 0 9
10	6 4 9 8	7 5 1 9	0 4 7 4	7 8 1 8	6 8 3 2	9 6 8 3	9 8 7 2	4 0 9 0
11	6 7 2 2	9 8 6 9	9 3 6 1	7 8 7 5	4 8 8 3	1 3 1 5	9 6 7 9	8 8 3 4
12	9 7 4 8	5 9 3 2	5 1 1 5	2 7 2 1	0 0 3 3	9 3 0 3	9 7 1 3	4 0 1 2
13	5 6 4 1	1 4 1 7	1 4 1 9	7 4 3 4	8 1 6 5	7 3 6 8	1 2 1 8	5 0 3 9
14	7 4 4 4	9 2 0 0	8 8 4 0	5 8 8 2	4 3 9 8	3 9 0 4	9 1 9 9	9 3 3 6
15	8 2 7 9	3 0 1 9	4 6 7 2	3 7 4 3	3 9 7 9	4 6 8 9	9 0 2 1	6 9 9 0
16	0 1 6 1	7 6 1 7	1 0 2 4	2 3 8 7	2 8 9 1	6 6 7 7	1 5 8 5	2 4 8 2
17	7 3 8 8	9 7 5 9	7 5 5 5	6 6 2 4	9 9 7 7	2 0 0 8	5 5 9 6	9 7 4 0
18	7 8 3 0	4 7 1 4	3 6 9 5	2 9 1 9	1 8 0 4	4 0 4 4	1 0 3 4	2 5 9 7
19	9 8 8 7	4 2 1 6	6 5 2 6	4 5 3 5	8 4 3 0	5 2 7 0	9 6 0 5	0 7 6 8
20	1 2 6 1	2 5 1 6	8 5 6 9	2 3 1 0	3 9 3 9	8 7 0 3	9 8 4 1	0 3 5 3
21	3 9 4 7	4 9 3 7	7 6 3 4	2 5 4 3	6 2 3 9	7 4 5 5	2 0 5 5	7 7 9 5
22	4 5 5 0	8 1 0 3	1 2 5 0	2 3 0 4	1 1 3 8	9 7 8 8	9 1 4 4	4 5 2 6
23	1 3 4 4	9 6 9 7	2 3 8 3	6 9 7 6	6 2 5 1	4 2 0 1	2 0 3 8	6 5 5 2
24	8 9 7 6	5 8 2 3	8 4 8 7	0 4 6 0	3 1 0 6	9 1 6 6	2 7 1 7	7 6 0 1
25	7 7 1 0	9 9 4 3	6 9 7 8	8 2 7 3	9 7 1 4	9 7 0 0	1 5 6 6	2 8 8 9
26	6 9 5 9	6 0 0 8	8 4 4 2	2 2 8 2	1 5 2 4	2 5 1 7	5 8 1 8	0 0 8 1
27	7 9 4 1	2 3 1 2	2 4 3 1	6 7 0 2	9 9 8 4	3 4 6 9	3 0 8 5	4 7 6 2
28	2 2 8 4	0 8 9 6	9 1 0 7	5 5 4 2	7 3 1 9	3 7 8 2	1 0 6 8	9 5 7 4
29	9 5 9 4	7 4 1 6	9 3 6 5	6 0 4 5	1 1 8 3	5 9 1 6	9 5 9 9	1 1 4 3
30	4 6 1 3	8 5 4 9	6 3 6 9	3 2 0 8	5 1 0 9	9 6 8 0	1 1 6 8	6 1 3 3

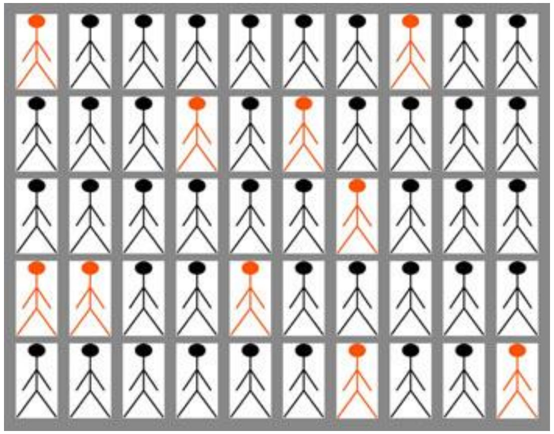
Sampling

Lottery method

- Population is finite i.e. number is relatively small and known
- Elements of population can be easily identified and numbered.

Example:

- 50 people in a class, need to draw a sample of 10 students.
- 50 numbers on paper in a hat, draw 10 numbers.



An example of simple random sampling of 10 subjects, represented by the red ‘stickmen’, selected at random from a total of 50 subjects using a lottery method.

Sampling

Table method

- Population is large
- Use tables of random numbers.
- Tables 7(a) to 7(c) in notes.

Example:

- Section of road to be tested: km 21,654 to 21,704, width is 3 meters.

- Start point: $P_s = 21,654$ km
  End point: $P_e = 21,704$ km
  Section length: $L_s = P_e - P_s = 21,704 - 21,654 = 0,050$ km.
  Width: $W_s = 3$ m

- From table 7(a): Row 20, Column 1(R_x) and column 2(R_y):

$$R_x = 0,0941$$

$$R_y = 0,8008$$

Sampling

Example (Continued):

- R_x represents random number for the width coordinate.
  R_y represents random number for the length coordinate.

- L = km length were sample must be taken
  $= P_s + R_yL_s$
  $= 21{,}654 + 0{,}8008(0{,}050)$
  $= 21{,}694$

- W = width = width from edge of road where sample must be taken
  $= R_xW_s$
  $= 0{,}0941(3)$
  $= 0{,}282$ m

- Next sample point:

  Same calculations, use row 21 in table 7(a)

Characteristics of Variability

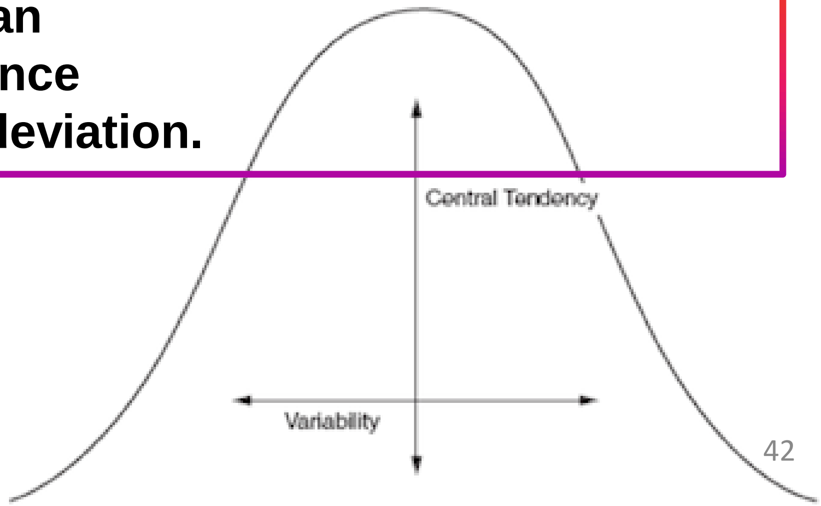
Variability:

- The extent to which data points in a statistical distribution or data set diverge from the average or mean value.

- Variability also refers to the extent to which these data points differ from each other.

- There are four commonly used measures of variability:

Range
Mean
Variance
Standard deviation.



Characteristics of Variability

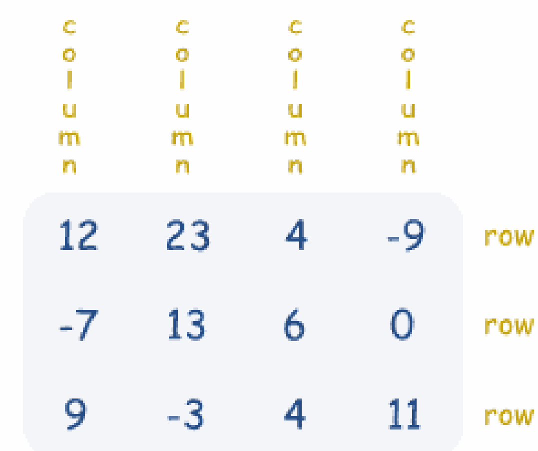
Frequency Distribution

Frequency:

Frequency refers to how often something occurs.

Array:

An array is an orderly arrangement of numbers, often in rows, columns or a matrix.



Array of Numbers
(3 rows x 4 columns)

Characteristics of Variability

Frequency Distribution

Frequency Distribution:

- A frequency distribution is an arrangement of the values that one or more variables take in a sample, usually in table format.
- Each entry (class or category) in the table contains the frequency or count of the occurrences of values within a particular group or interval.
- The table summarizes the distribution of values in the sample.

Scores:
1,1,2,2,2,2,2,3,3,3,3,4,4,5

Score	Frequency
1	2
2	5
3	4
4	2
5	1

Characteristics of Variability

Frequency Distribution

Example:

The following values are final marks in a test taken by 80 students:

68	84	75	82	68	90	62	88	76	93
73	79	88	73	60	93	71	59	85	75
61	65	75	87	74	62	95	78	63	72
66	78	82	75	94	77	69	74	68	60
96	78	89	61	75	95	60	79	83	71
79	62	67	97	78	85	76	65	71	75
65	80	73	57	88	78	62	76	53	74
86	67	73	81	72	63	76	75	85	77

Characteristics of Variability

Frequency Distribution

- The highest mark is 97 and the lowest mark is 53.
- We can say the range is : 97 – 53 = 44.
- To determine the class interval: 44 / 10 = 4,4.
- Set the class interval as 5.
- Class intervals: [50,54], [55,59], [60,64], [65,69], ...
- [50,54] is a closed interval, contains values:

  {50,51,52,53,54}

- Class intervals do not overlap, accommodate all the data and are of the same size.

Characteristics of Variability

Frequency Distribution

Class interval	Student marks (Mathematics)
50–54	53
55–59	59 57
60–64	62 60 61 62 63 60 61 60 62 62 63
65–69	68 68 65 66 69 68 67 65 65 67
70–74	73 73 71 74 72 74 71 71 73 74 73 72
75–79	75 76 79 75 75 78 78 75 77 78 75 79 79 78 76 75 78 76 76 75 77
80–84	84 82 82 83 80 81
85–89	88 88 85 87 89 85 88 86 75
90–94	90 93 93 94
95-99	95 96 95 97

Characteristics of Variability

Frequency Distribution

Re-arrange the marks in each class in ascending order (an ordered array):

Class interval	Student marks (Mathematics)
50–54	53
55–59	57 59
60–64	60 60 60 61 61 62 62 62 62 63 63
65–69	65 65 65 66 67 67 68 68 68 69
70–74	71 71 71 72 72 73 73 73 73 74 74 74
75–79	75 75 75 75 75 75 75 76 76 76 76 77 77 78 78 78 78 78 79 79 79
80–84	80 81 82 82 83 84
85–89	85 85 85 86 87 88 88 88 89
90–94	90 93 93 94
95-99	95 95 96 97

Characteristics of Variability

Frequency Distribution

Characteristics of Variability

Frequency Distribution

Conclusions out of the frequency distribution:

- **The highest mark is 97**
- **The lowest mark is 53**
- **The range is 97 – 53 = 44**
- **The five highest ranking students have marks {97, 96, 95, 95, 94}**
- **The five lowest ranking students have marks {53, 57, 59, 60, 60}**
- **The mark of the student ranking tenth highest is 88**
- **The number of students receiving marks of 75 or higher is 44**
- **The number of students receiving marks below 85 is 63**
- **The marks which did not appear are**

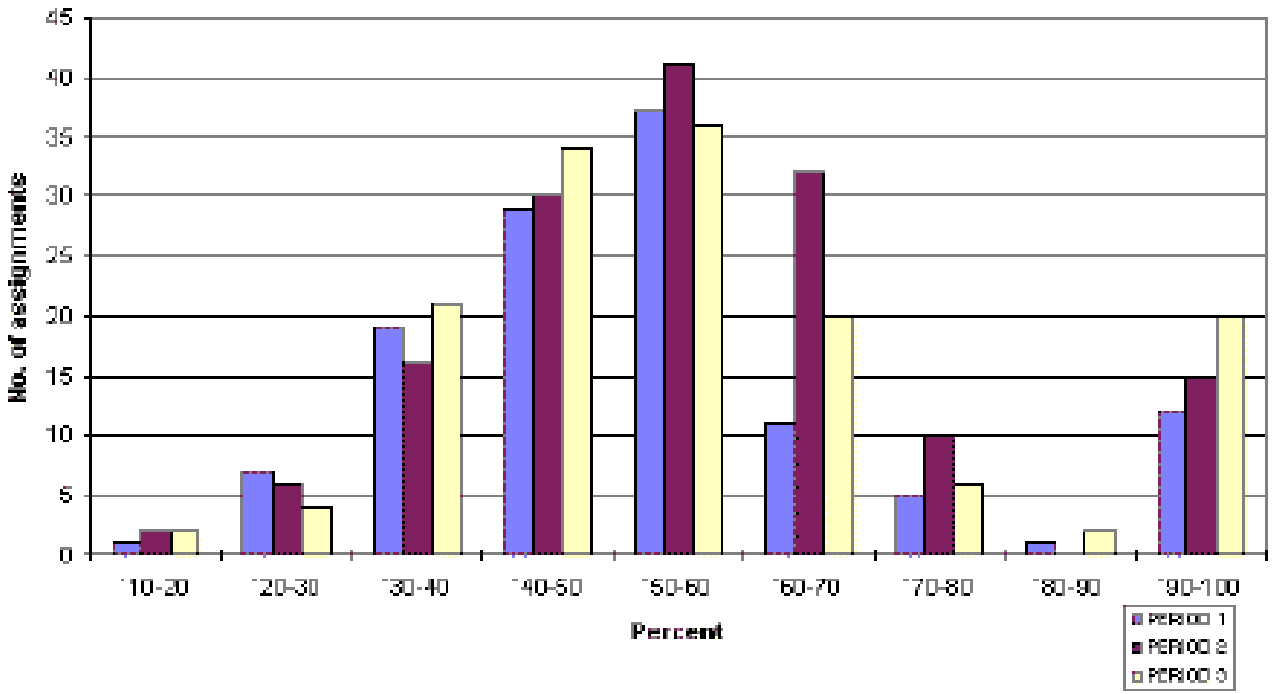
{0–52, 54-56, 58, 64, 70, 91, 92, 98-100}

Characteristics of Variability

Histogram

Frequency Histogram:

- **Defined as a graphical representation of data.**
- **The data is grouped into ranges (such as "40 to 49"), and then plotted as bars.**
- **Similar to a Bar Graph, but each bar represents a range of data.**

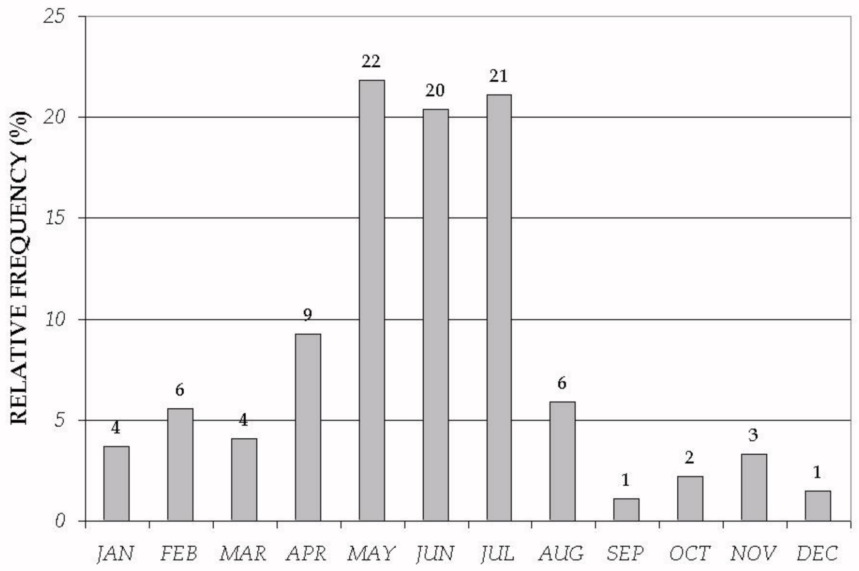


Characteristics of Variability

Histogram

Relative Frequency Histogram:

- **Shows the proportion of events rather than the absolute number of events.**
- **The relative frequency of a class is the frequency of the class divided by the total number of frequencies of the class and is generally expresses as a percentage.**
- **The sum of the relative frequencies of all classes is 1 or 100 %.**



Characteristics of Variability

Histogram

Cumulative Frequency Distribution:

- The total of a frequency and all frequencies below it in a frequency distribution.
- It is the 'running total' of frequencies.

Example:

Consider the following table populated with the weight of a hundred students:

Mass (kg)	Frequency (number of students)
60-62	5
63-65	18
66-68	42
69-71	27
72-74	8
	Total 100

Characteristics of Variability

Histogram

Example (Continued):

- **The cumulative frequency up to and including the class interval 66-68 is:**

$$5 + 18 + 42 = 65$$

- **65 students have masses less than 68,5 kg**

Cumulative Frequency Table of previous data:

Mass (kg)	Frequency (number of students)
less than 59,5	0
less than 62,5	5
less than 65,5	23
less than 68,5	65
less than 71,5	92
less than 74,5	100

Characteristics of Variability

Histogram

Example:

The following data set contains 80 compressive strengths of aluminium-lithium alloy specimens:

105	221	183	186	121	181	180	143
97	154	153	174	120	168	167	141
245	228	174	199	181	158	176	110
163	131	154	115	160	208	158	133
207	180	190	193	194	133	156	123
134	178	76	167	184	135	229	146
218	157	101	171	165	172	158	169
199	151	142	163	145	171	148	158
160	175	149	87	160	237	150	135
196	201	200	176	150	170	118	149

Characteristics of Variability

Histogram

Example (Continued):

- The following distribution table contains nine classes, each of width 20 units:

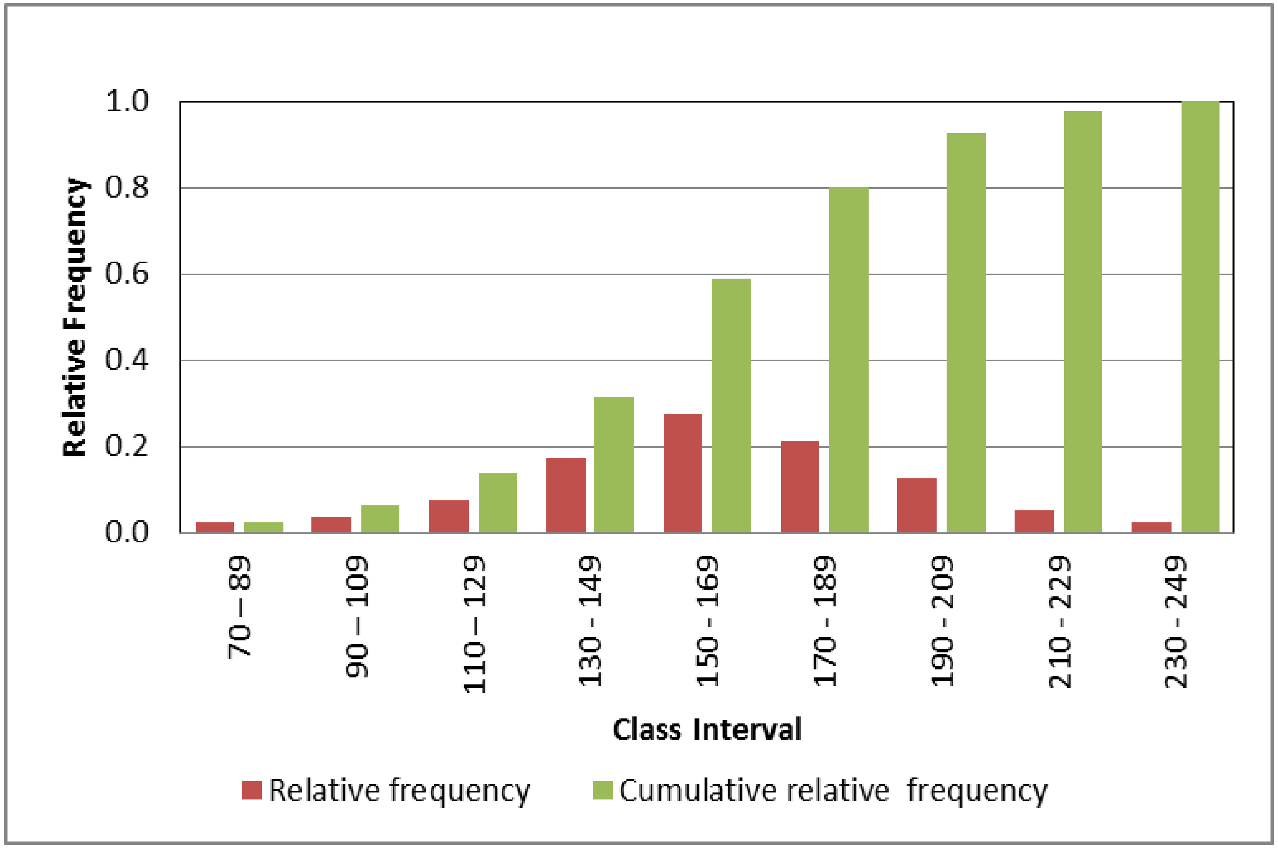
Class interval (psi)	Tally	Observed Frequency	Relative frequency	Cumulative relative frequency
70 – 89	||	2	0,0250	0,0250
90 – 109	|||	3	0,0375	0,0625
110 – 129	~~||||~~ |	6	0,0750	0,1375
130 - 149	~~||||~~ ~~||||~~ ||||	14	0,1750	0,3125
150 - 169	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ||	22	0,2750	0,5875
170 - 189	~~||||~~ ~~||||~~ ~~||||~~ ||	17	0,2125	0,8000
190 - 209	~~||||~~ ~~||||~~	10	0,1250	0,9250
210 - 229	||||	4	0,0500	0,9750
230 - 249	||	2	0,0250	1,0000

Characteristics of Variability

Histogram

Example (Continued):

- **The following depicts the relative and cumulative relative frequency of the previous data:**

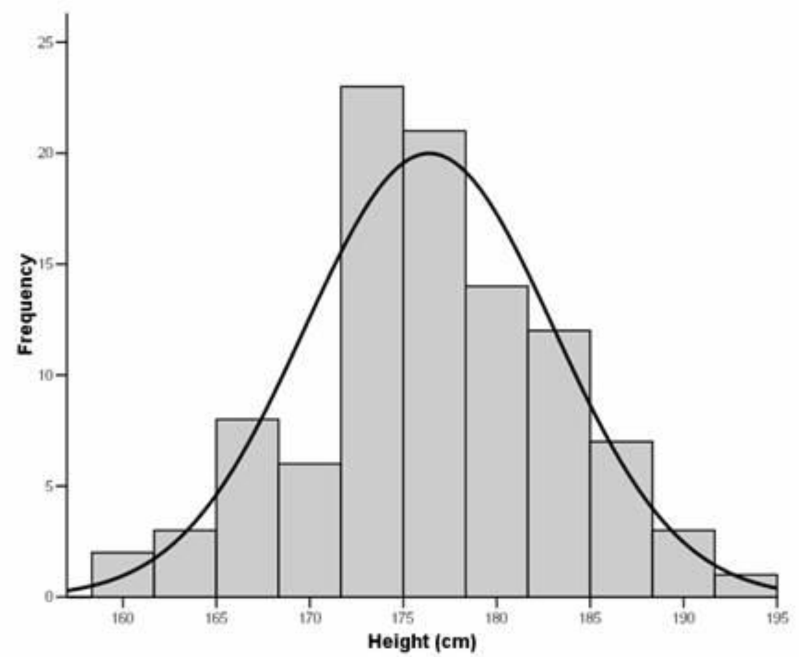


Characteristics of Variability

Normal Distribution

Normal distribution:

- **Follows a "Bell" curve.**
- **The centre contains the greatest number of a value (Mean) and therefore is the highest point on the arc of the line.**
- **The curve is concentrated in the centre and decreases on either side.**



Characteristics of Variability

Normal Distribution

Normal distribution (Continued):

- **The bell curve signifies that the data is symmetrical and thus we can create reasonable expectations as to the possibility that an outcome will lie within a range to the left or right of the centre, once we can measure the amount of deviation (σ) contained in the data.**

- **The mean (μ) identifies the position of the centre and the standard deviation (σ) determines the height and width of the bell.**

- **A large standard deviation creates a bell that is short and wide while a small standard deviation creates a tall and narrow curve.**

- **The mean and the median are the same in a normal distribution.**

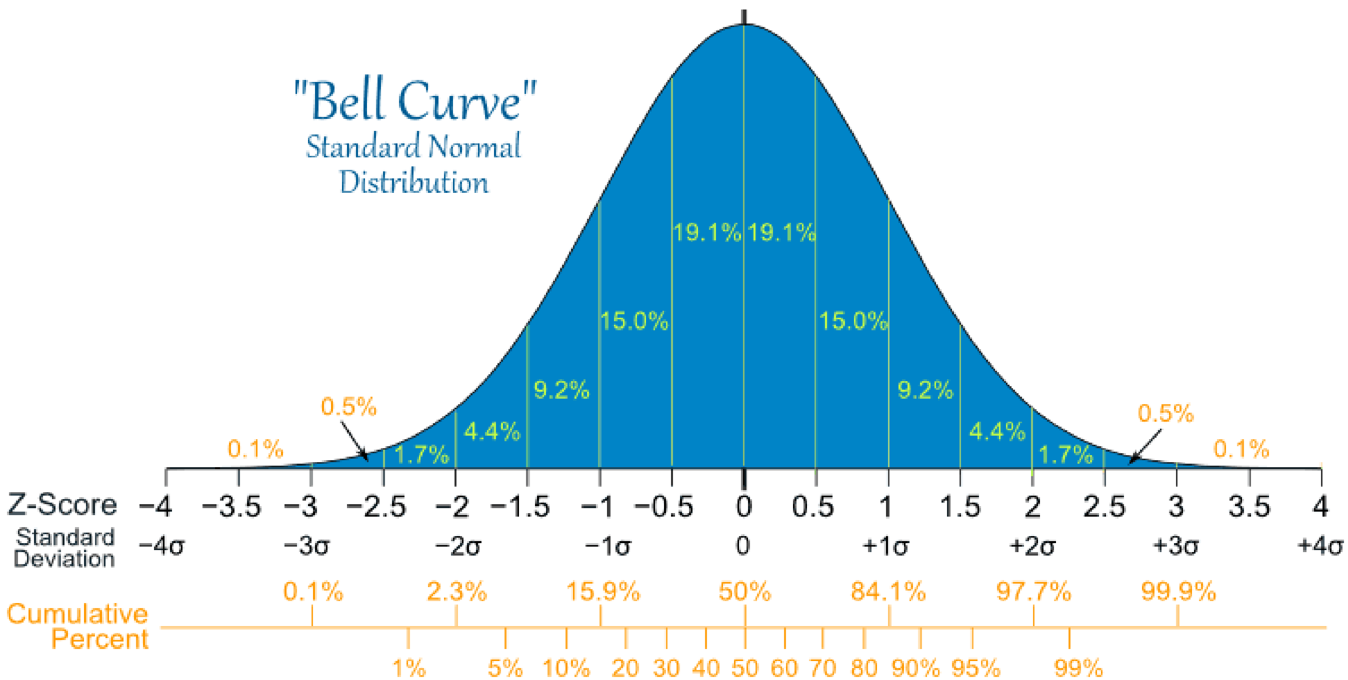
- **The area under the curve is equal to one.**

Characteristics of Variability

Normal Distribution

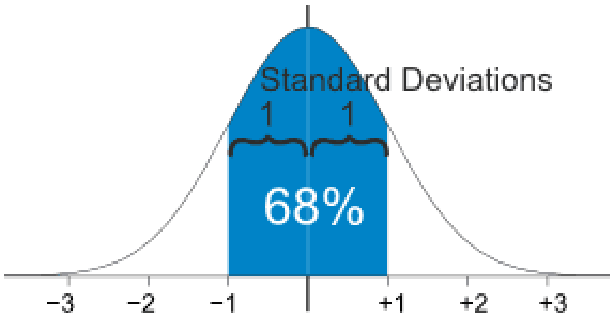
Normal distribution (Continued):

- **The total area under the curve is equal to 1 (100%).**
- **About 68% of the area under the curve falls within 1 standard deviation.**
- **About 95% of the area under the curve falls within 2 standard deviations.**
- **About 99.7% of the area under the curve falls within 3 standard deviations.**

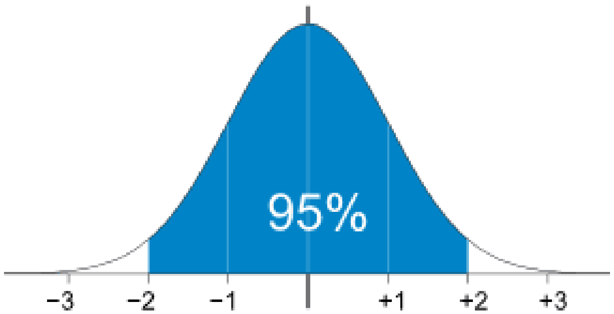


Characteristics of Variability

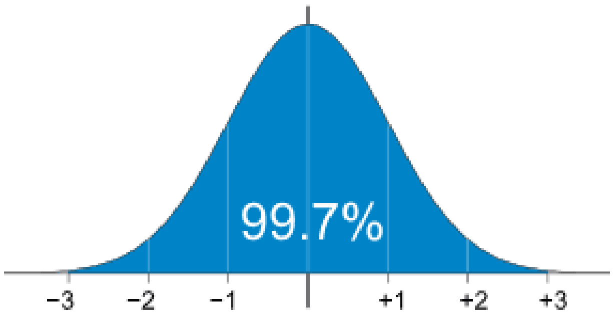
Normal Distribution



68% of values are within 1 standard deviation of the mean



95% of values are within 2 standard deviations



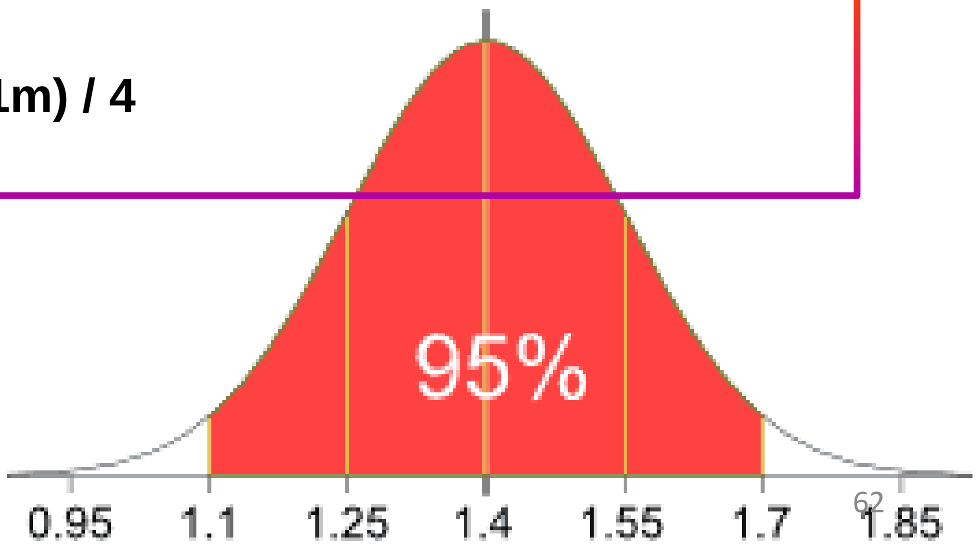
99.7% of values are within 3 standard deviations

Characteristics of Variability

Normal Distribution

Example:

- 95% of students at school are between 1.1m and 1.7m tall.
- Assuming this data is normally distributed the mean and standard deviation can be calculated:
- The mean is halfway between 1.1m and 1.7m:
- Mean = (1.1m + 1.7m) / 2 = 1.4m
- 95% is 2 standard deviations either side of the mean (a total of 4 standard deviations) so:
- 1 standard deviation = (1.7m-1.1m) / 4
  = 0.6m / 4 = 0.15m

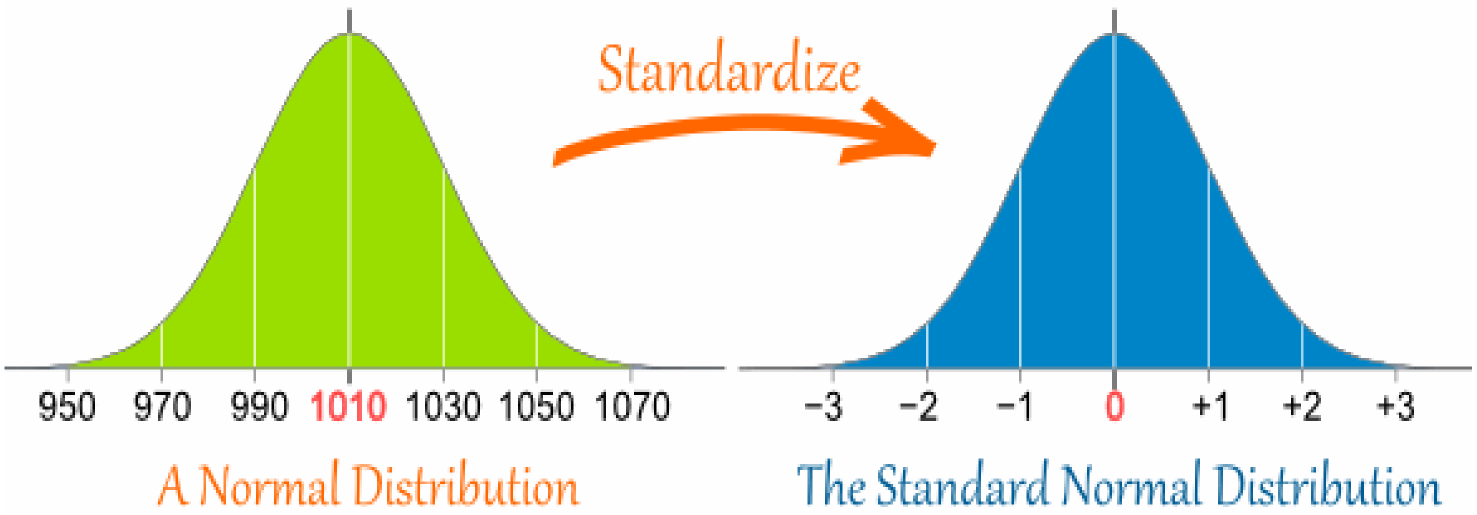


Characteristics of Variability

Normal Distribution

Standard score:

- **The number of standard deviations from the mean is also called the "Standard Score", "sigma" or "z-score“.**
- **How to convert a value to a Standard Score ("z-score"):**
  - **first subtract the mean,**
  - **then divide by the Standard Deviation**
- **And doing that is called "Standardizing":**

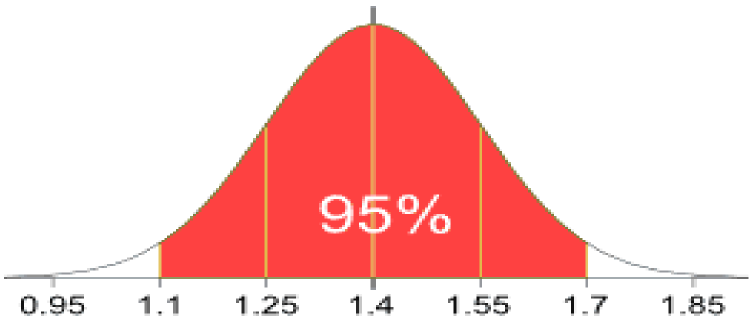


Characteristics of Variability

Normal Distribution

Example:

In the previous example in that same school one of your friends is 1.85m tall.



- **Out of the bell curve it can be seen that 1.85m is 3 standard deviations from the mean of 1.4, so:**
  - **Your friend's height has a "z-score" of 3.0**
- **It is also possible to calculate how many standard deviations 1.85 is from the mean.**
- **How far is 1.85 from the mean?**
  - **It is 1.85 - 1.4 = 0.45m from the mean**
- **How many standard deviations is that?**
- **The standard deviation is 0.15m, so:**
  - **0.45m / 0.15m = 3 standard deviations**

Characteristics of Variability

Normal Distribution

Example:

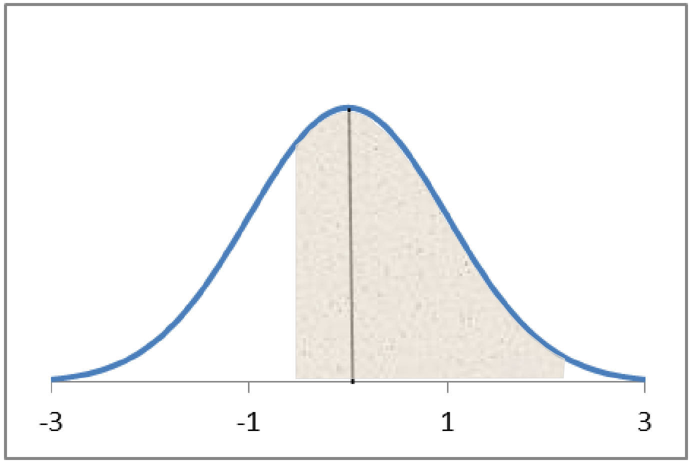
- **How to use table 8 in the notes:**
- **What is the Percentage of Population Between 0 and 0.45?**
- **In table 8 is the "0.1"s running down, and then the "0.01"s running along (to the right).**
- **Start at the row for 0.4, and read along until 0.45: there is the value 0.1736, this is the area under the curve between 0 and 0.45.**
- **So 0.1736 of the population are between 0 and 0.45 Standard Deviations from the Mean.**
- **And 0.1736 is 17.36%.**
- **So 17.36% of the population are between 0 and 0.45 Standard Deviations from the Mean.**

Characteristics of Variability

Normal Distribution

Example:

- **Find the area under the normal curve between z = -0,46 and z = 2,21**



- **Area = Area [z: -0,46 0] + [z: 0 2,21]**
  = Area [z: 0,46 0] + [z: 0 2,21]
  = 0,1772 + 0,4864
  = 0,6636

- **The result 0,6636 is the required area and represents the probability that z is between -0,46 and 2,21 , denoted by:**

$$\Pr\{ -0{,}46 \leq z \leq 2{,}21\}$$

Characteristics of Variability

Standard Error:

- **Represents how well the sample mean approximates the population mean, i.e. the standard error gives a measure of how well a sample represents the population.**
- **The larger the sample, the smaller the standard error, and the closer the sample mean approximates the population mean.**
- **When the sample is representative, the standard error will be small.**
- **The standard error is the standard deviation divided by the square root of N (the sample size):**

$$\text{Standard Error } (s_{\bar{x}}) = \frac{s}{\sqrt{n}}$$

where:

S = Standard Deviation (sample)
n = quantity of numbers in group

Characteristics of Variability

Example:

- Consider the following sample sets:

	Sample 1	Sample 2	Sample 3	Sample 4
	9	6	5	8
	2	6	3	1
	1	8	6	7
		8	4	1
		3	7	3
		8	2	3
			6	4
			9	7
			7	1
			1	8
			1	9
			7	9
				3
				1
				6
				8
				3
				4
Mean:	4	6.5	4.83	4.78
Std dev, s:	4.36	1.97	2.62	2.96
Sample size, n:	3	6	12	18
sqrt(n):	1.73	2.45	3.46	4.24
Standard error, s/sqrt(n):	2.52	0.81	0.76	0.70

- Note how the standard error reduces with increasing sample size

Characteristics of Variability

Confidence Level:

- **The confidence level tells you how sure you can be .**
- **Confidence level is a percentage, typically 90%, 95% or 99%.**
- **The higher the level of confidence, the wider the interval.**
- **The lower the level of confidence, the shorter the interval.**
- **A confidence level of 95% means that 19 out of 20 times the confidence interval around the population mean μ is right.**

For a confidence level of 95%:

Confidence interval:

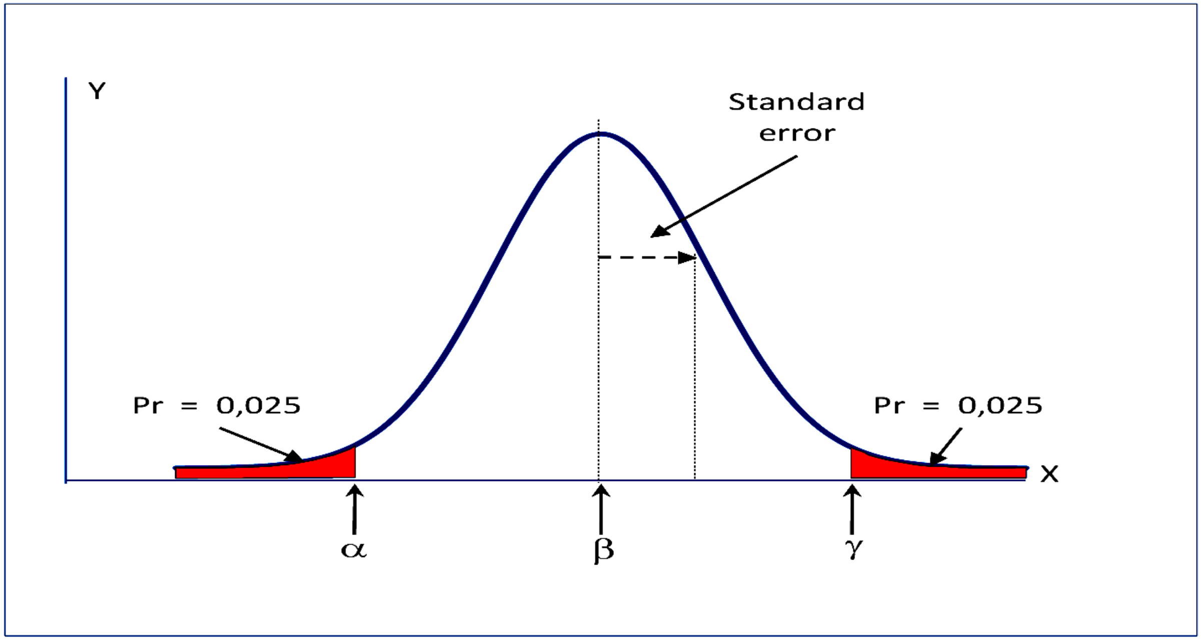
$$\mu = \bar{x} \pm z_{0,025} \frac{\sigma}{\sqrt{n}}$$

Characteristics of Variability

Confidence Interval:

A confidence interval is a range around a measurement that conveys how precise the measurement is.

- **For a confidence level of 95%, select the smallest range under the normal distribution of $\bar{x}$ that will enclose a probability of 95%.**
- **This leaves 2,5% probability excluded under half the normal distribution.**
- **For this we need a z-value of 1.96 (Table 8: 0.5 -0.025 = 0.475)**



Characteristics of Variability

Example:

Sixteen marks were sampled randomly from a very large class that had a standard deviation of 12; these 16 marks had a mean of 58. Find a 95% confidence interval for the mean mark of the whole class.

$$n = 16$$
$$\sigma = 12$$
$$\bar{x} = 58$$

$$\mu = \bar{x} \pm z_{0,025} \frac{\sigma}{\sqrt{n}}$$

$$= 58 \pm (1{,}96)\frac{12}{\sqrt{16}}$$

$$= 58 \pm 6$$

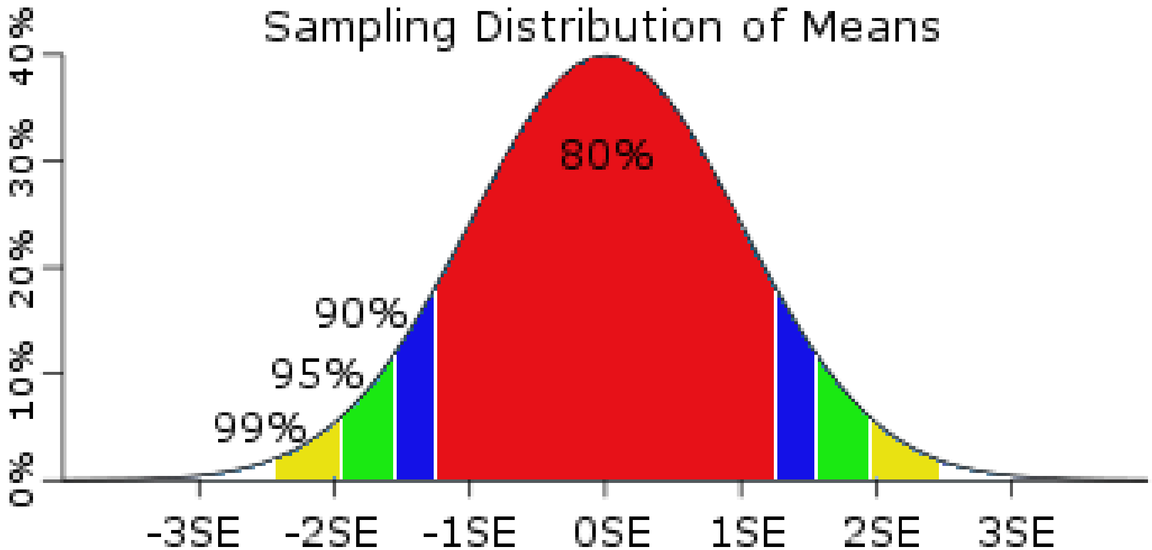
The 95% confidence interval for μ is:

$$52 < \mu < 64$$

Characteristics of Variability

For other confidence intervals:

Confidence level	Standard Errors from mean
99%	2.58
95%	1.96
90%	1.64
80%	1.28



Statistical Hypothesis

Hypothesis:

- A hypothesis is an explanation for a set of observations.
- Most hypothesis are either "If, then" statements or else forms of the null hypothesis.

Null Hypothesis:

- A proposition that undergoes verification to determine if it should be accepted or rejected in favour of an alternative proposition.
- Often the null hypothesis is expressed as "There is no relationship between two quantities.”

Example:

- In market research, a null hypothesis would be "A ten-percent increase in price will not adversely affect the sale of this product."
- Based on survey results, this hypothesis will be either accepted as correct or rejected as incorrect.

Statistical Hypothesis

Type I Error:

- False positive.
- Usually a type I error leads one to conclude that a thing or relationship exists when really it doesn't.

Example of Type I Error:

A patient has a disease for which he is being tested when really the patient does not have the disease.

Type II Error:

- False negative.
- Usually a type II error leads one to conclude that a thing or relationship which is perceived as not existing, actually exists.

Example of Type II Error:

A blood test failed to detect the disease it was designed to detect, in a patient who really has the disease.

Statistical Hypothesis

		Reality	
		True	False
Measured / Perceived	True	Correct ☺	Type I False Positive
	False	Type II False Negative	Correct ☺

Central Limit Theory

Central Limit Theory:

- The Central Limit Theorem describes the characteristics of the "population of the means“, which has been created from the means of an infinite number of random population samples of size (N), all of them drawn from a given "parent population“.

- i.e. :
  Let the set of sample means be

$$\bar{X} = \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_j, \ldots, \bar{x}_\infty\}$$

  and the set of sample standard deviations be

$$S = \{s_1, s_2, s_3, \ldots, s_j, \ldots, s_\infty\}$$

- ***The Central Limit Theory states regardless of the distribution of the population, the distribution of $\bar{X}$ will tend to follow a normal curve as the sample size increases (>30).***

Tests of Significance

t – distribution:

- The t-distribution is generally used to estimate the uncertainty about the population mean μ.
- Sample size is usually small.
- It is symmetrical, bell-shaped, and similar to the standard normal curve.
- It differs from the standard normal curve, however, in that it has an additional parameter, called degrees of freedom (*v*), which changes its shape.

If $\bar{x}$ is the mean of a random sample of size n taken from a normal population having the mean μ and the variance σ^2, then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

With $v = n - 1$

Tests of Significance

Example:

A manufacturer of fuses claims that with a 20% overload, the fuses will blow in 12,40 minutes on the average.
To test this claim, a sample of 20 fuses was subjected to a 20% overload, and the times it took them to blow had a mean of 10,63 minutes and a standard deviation of 2,48 minutes.

If it can be assumed that the data constitute a random sample from a normal population, do they tend to support or refute the manufacturer's claim?

$$t = \frac{|\bar{x} - \mu|}{s/\sqrt{n}} = \frac{|10,63 - 12,40|}{2,48/\sqrt{20}} = 3,19$$

- **v = 20 – 1 = 19**
- **Out of table 9: probability that t will exceed 2,86 is 0,005**
- **t = 3,19 is more than 2,86; 0,005 is a very small probability.**
- **In all probability the mean blowing time of the fuses with a 20% overload is less than 12,40 minutes. (difference is significant)**

Tests of Significance

Example:

Consider the following results on ten batches of asphalt to determine the bitumen content, as tested by two laboratories:

Batch no.	1	2	3	4	5	6	7	8	9	10
Lab A	5.5	6.6	5.7	4.2	6.8	5.2	7.2	4.7	5.9	5.3
Lab B	6.1	6.6	5.8	4.8	6.5	5.9	7.2	5.0	6.0	5.2

Conduct the analysis in terms of the discrepancies between the laboratories:

Batch no.	1	2	3	4	5	6	7	8	9	10
Discrepancy	0.6	0.0	0.1	0.6	-0.3	0.7	0.0	0.3	0.1	-0.1

Tests of Significance

Example (Continued):

The mean value of the discrepancies is:

$$\bar{x} = \frac{0,6 + 0,0 + 0,1 + 0,6 + (-0,3) + 0,7 + 0,0 + 0,3 + 0,1 + (-0,1)}{10}$$

$$= \frac{2}{10}$$

$$= 0,20$$

The standard deviation is: square root (square of discrepancy – mean)

$$s = \sqrt{\frac{0,16 + 0,04 + 0,01 + 0,16 + 0,25 + 0,25 + 0,04 + 0,01 + 0,01 + 0,09}{10 - 1}}$$

$$= 0,35$$

Tests of Significance

Example (Continued):

$$t = \frac{|0{,}20 - 0|}{0{,}35/\sqrt{10}}$$

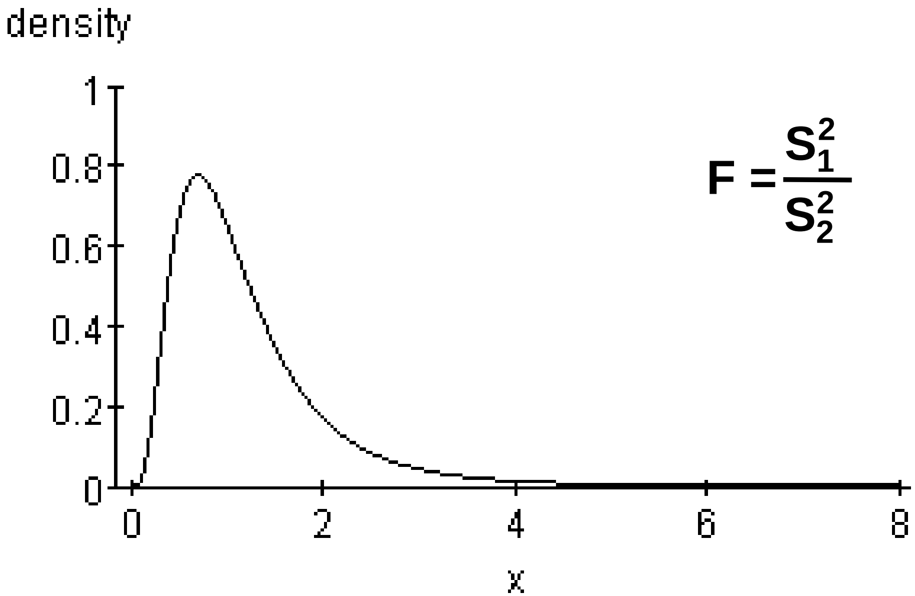
$$= 1{,}81$$

- **Degree of freedom = v = 10 – 1 = 9**
- **From table 9: I $t_{9;0,025}$ I = 2,26**
- **The calculated value of *t* is less than the value that would be exceeded by chance only 5% of the time.**
- **The difference between laboratories is not significant.**

Tests of Significance

F-distribution:

- t-test is used to compare two means, F-test is based on the ratio between two variances, i.e. compare the variance of two samples for significance.
- F-values are all positive.
- The F-distribution is not symmetrical.
- The F-distribution is skewed to the right and has a minimum value of 0, but no maximum value.
- F values depend on 2 parameters: numerator degrees of freedom (v_1) and denominator degrees of freedom (v_2).



$$F = \frac{S_1^2}{S_2^2}$$

Tests of Significance

Example:

- Laboratory A and Laboratory B are required to test 25 concrete cubes representative of the same batch at a certain date.
- The purpose of the exercise was to check whether the variability that could be assigned to the test results is different at a 5 % level of significance.

- Data obtained from the laboratories:

Laboratory	A	B
mean	32,0	36,2
S.D.	3,35	2,43
n	25	25

- 5% level of significance: table 10(a) for $f_{0,05\ v1\ v2}$
- Degrees of freedom: v1 = v2 = 25 – 1= 24
- $S_A^2 = 3,35^2$
- $S_B^2 = 2,43^2$

Tests of Significance

Example (Continued):

From table 10(a): $F_{0,05\ 24;24} = 1{,}98$

and

$$F = \frac{S_A^2}{S_B^2} = \frac{3{,}35^2}{2{,}43^2} = 1{,}90$$

$F < F_{0,05\ 24;24}$ which means that the F value is not significant, the two variances are equal.

Tests of Significance

Chi-square distribution:

- Chi-square (X^2)is a statistical test commonly used to compare observed data with data we would expect to obtain according to a specific hypothesis.
- The chi-square test is always testing what scientists call the null hypothesis, which states that there is no significant difference between the expected and observed result.
- Chi-square requires that you use numerical values, not percentages or ratios.
- The chi-square is the sum of the squared difference between observed (*o*) and the expected (*e*) data, divided by the expected data in all possible categories:

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Where:
O represents the observed frequency
E represents the expected frequency

Tests of Significance

Example:

In 200 tosses of a coin, 115 heads and 85 tails were observed. We are required to test the hypothesis that the coin is fair.

- **Observed frequencies are:**

Heads: $O_1 = 115$
Tails: $O_2 = 85$

- **Expected frequencies are:**

Heads: $E_1 = 100$
Tails: $E_2 = 100$

- **Then** $$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(115 - 100)^2}{100} + \frac{(85 - 100)^2}{100}$$

=4,50

Tests of Significance

Example (Continued):

- **Number of categories or classes is 2**
- **Hence degrees of freedom, *v*, is 1**
- **Out of Table 12:** $\chi^2_{.95}$ **= 3,84**
- **Since 4,50 > 3,84**

We reject the hypothesis that the coin is fair at the 95% level of significance.

- **However, at the critical value of** $\chi^2_{.99}$ **for *v* = 1 is 6,63 and since**

4,50 < 6,63

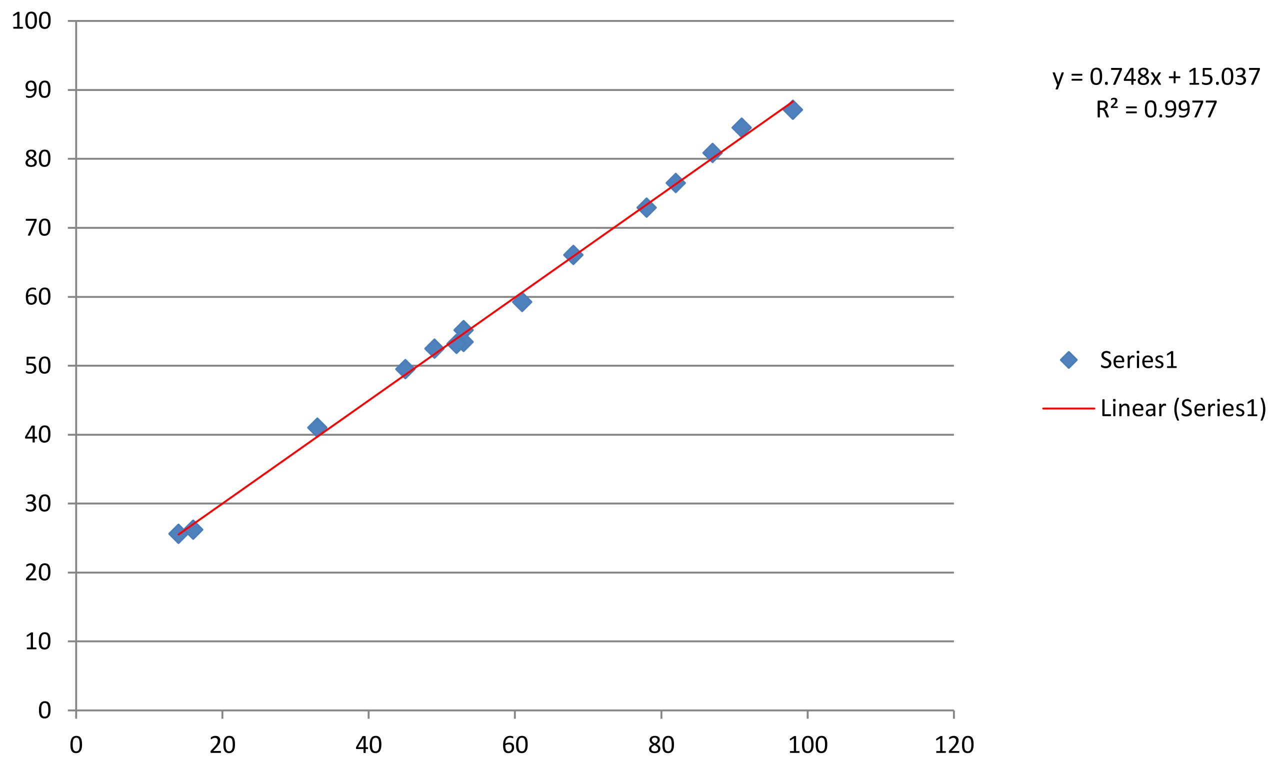
we cannot reject the hypothesis that the coin is fair at this level of confidence.

Regression Analysis

- Regression analysis is a statistical tool for the investigation of relationships between variables.
- Regression analysis is widely used for prediction and forecasting

Example:

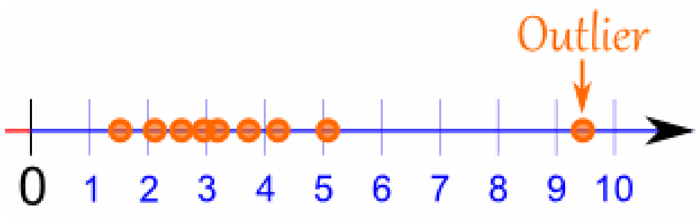
x	y
53	53.42
91	84.53
87	80.87
49	52.45
14	25.6
98	87.12
78	72.93
82	76.5
53	55.14
33	41
45	49.51
52	53.16
16	26.22
68	66.04
61	59.26



Outliers

Outliers:

- **Outliers are extreme cases which can strongly influence statistical calculations.**
- **An outlier can be a value that "lies outside" (is much smaller or larger than) most of the other values in a set of data.**



- **To determine whether a suspicious value is an outlier, the status of the standard deviation available has to be known.**

$$C_o = |x_o - \bar{x}|$$

- **Where: x_o = the suspicious value in a set of observations**
  $\bar{x}$ = mean of the set of observations.

- **If $C_o > C_{0,01;\,n}\,\sigma$ (see Table 13)**

the suspicious value x_o, is regarded as an outlier and should be excluded from the set of data.

Outliers

Example:

- **Let the dataset**

{X: 5,1 5,2 5,2 5,2 5,0 5,6}

represent percentage bitumen contents. The mean is 5,22 and the sample standard deviation s is 0,20. Determine whether the suspicious value 5,6 is an outlier.

- **The population standard deviation for percentage bitumen content is 0,3% (table 11(a)).**

$$\bar{x} = 5,2$$

$$x_o = 5,6$$

$$\sigma = 0,3$$

$$C_o = |x_o - \bar{x}| = |5,6 - 5,2| = 0,4$$

- **and**

$$C_{0,01;\,6}\,\sigma = (2,68)(0,3) = 0,8 \text{ (Table 13, n = 6)}$$

- **hence** $C_o < C_{0,01;\,6}$
- **and the suspicious value** x_o **is NOT an outlier.**

Uncertainty of Measurement

Measurement:

- A measurement tells us about a property of something.
- A measurement tells us how heavy an object is, or how hot, or how long it is.
- A measurement gives a number to that property.
- The result of a measurement is normally in two parts: a number and a unit of measurement, e.g. “How long is it? ... 2 metres.”

Uncertainty of Measurement:

- Uncertainty of measurement is the doubt that exists about the result of any measurement.
- For every measurement - even the most careful - there is always a margin of doubt.
- This might be expressed as “give or take” ... e.g. a stick might be two metres long “give or take a centimetre”.
- In order to quantify an uncertainty the width of the margin, or interval, and the confidence level (which states how sure we are that the “true value” is within that margin) are needed.

Uncertainty of Measurement

Example:

The measurement is: 5,07 g ± 0,02 g.

This implies that the experimenter is confident that the actual value for the quantity being measured lies between 5,05 g and 5,09 g.

Always round the experimental measurement or result to the same decimal place as the uncertainty.

Wrong: 52,3 cm ± 4,15 cm
Correct: 52 cm ± 4 cm

Uncertainty of Measurement

- If the ranges of two measured values don't overlap, the measurements are *discrepant* (the two numbers don't agree).

- If the ranges overlap, the measurements are said to be *consistent*

Measurements don't agree	0,86 s ± 0,02 s and 0,98 s ± 0,02 s
Measurements agree	0,86 s ± 0,08 s and 0,98 s ± 0,08 s

Uncertainty values associated with a number of tests on road building materials are given in Table 11 (a) to (c)

Uncertainty of Measurement

Error:

- Error is the difference between the measured value and the ‘true value’ of the thing being measured.
- Error can be dealt with, as it is usually the result of something the person who is conducting the experiment is doing or as a result of something that is happening with the equipment.
- Whenever possible, one strives to correct for any known errors. One can do so, e.g., by applying corrections from calibration certificates.

Uncertainty:

- Uncertainty is a quantification of the doubt about the measurement result.
- Uncertainty is something that is inherent in the experiment and cannot be eliminated., it can only be characterised.
- Any error whose value we do not know is a source of uncertainty.
- Uncertainty cannot be avoided but it can be reduced by using 'better' apparatus.

Uncertainty of Measurement

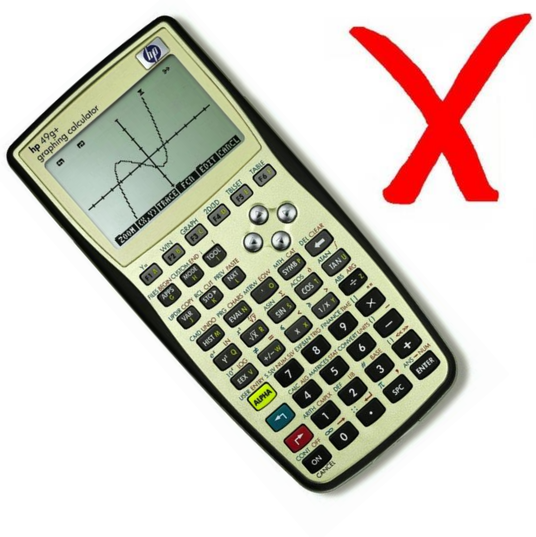
Repeatability:

The difference between two test results obtained by the same operator with the same apparatus under constant operating conditions on identical test material.

Reproducibility:

The difference between two single and independent test results obtained by different operators working in different laboratories on identical test material

Please bring a Calculator for the workshop
Avoid HP programmable calculators, bring a Casio or Sharp scientific calculator

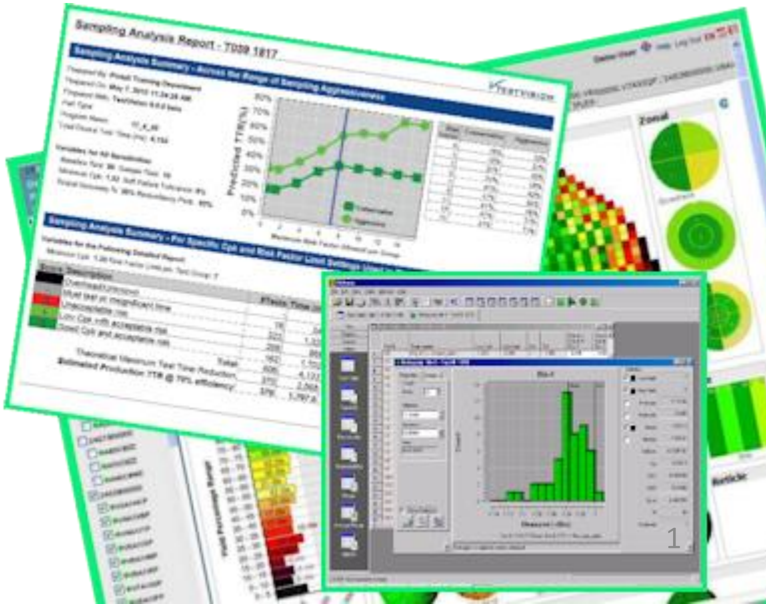


End of Chapter 2

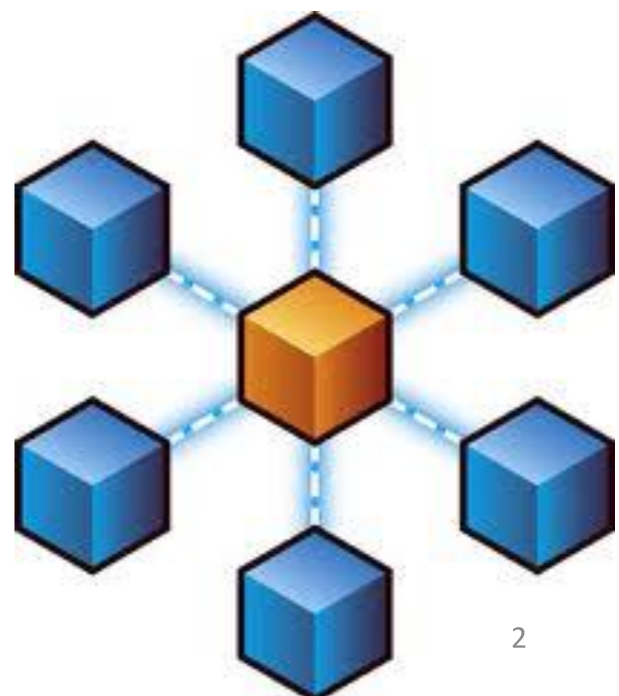
Assessment and Analysis of Test Data

Presented by SARF

Presenter:
Ron Berkers



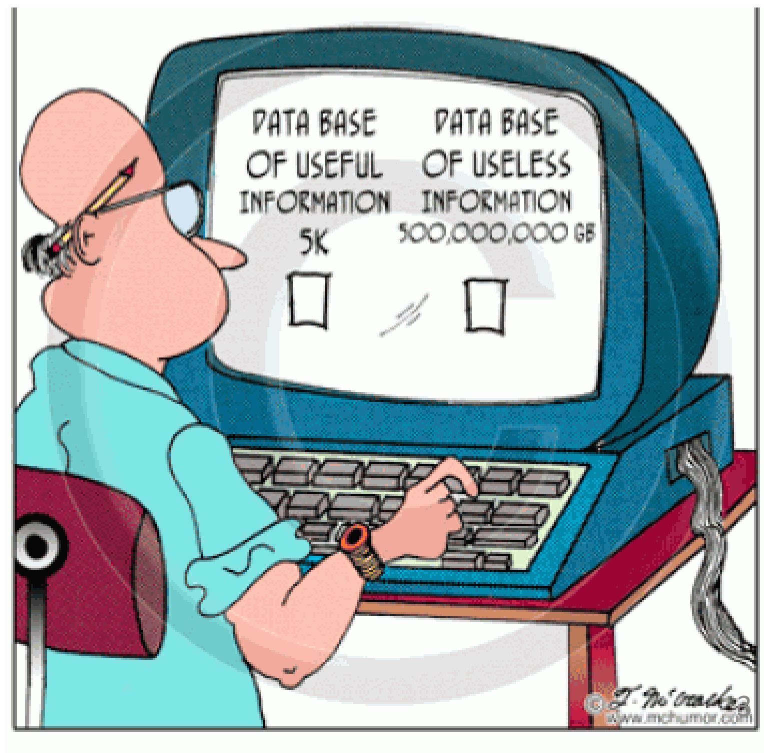
Module C
Data Management



Data

Data:

- **Data means a set of quantities, figures and statistics, relating to any event or product.**
- **A set of data can be either numerical or non-numerical.**
- **Numeric data are numbers (like age, cost, mass, etc.).**
- **Non-numeric data are not real numbers (like name, address, etc.).**



Data Collection

Data Collection:

- **The process of gathering and measuring information on variables of interest.**
- **Data collection is done in an established, systematic fashion that enables one to answer routine data management questions, test hypotheses, and evaluate outcomes.**

Data Collection

Data Integrity:

- **Accurate data collection is essential to maintaining the integrity of data quality.**
- **Both the selection of appropriate data collection methods and clearly delineated instructions for their correct use reduce the likelihood of errors occurring.**

Data Collection

Data Quality Assessment:

- **Data quality assessment is a multi-step procedure for determining according to prescribed procedures whether or not a data set is suitable for its intended purpose.**
- **The assessment is a statistical evaluation of data to determine if it is of the type, quantity, and quality needed.**
- **Data quality assurance and quality control are two approaches that can preserve data integrity and ensure the scientific validity of the investigation or control results.**
- **Quality assurance activities take place before data collection begins.**
- **Quality control activities take place during and after data collection.**

Data Collection

Data Quality Assurance:

- **The main focus of data quality assurance is prevention.**
- **Data quality assurance tries to forestall problems with data collection.**
- **An important component of quality assurance is developing a rigorous and detailed training plan.**

Data Collection

Data Quality Control:

- Quality control activities such as the monitoring, detection and actions) occur during and after data collection.
- A clearly defined communication structure is a necessary pre-condition for establishing monitoring systems.
- While site visits may not be appropriate in all applications, failure to perform regular audit records, whether quantitative or qualitative, will make it difficult for investigators to verify that data collection is proceeding according to standard procedures.

Examples of data collection problems that require prompt action include:

- Errors in individual data items.
- Systematic errors.
- Violation of protocol.
- Problems with individual staff or site performance.
- Fraud or scientific misconduct.

Data Collection

Data reviewing prior to analysis:

Steps:

- **Thorough review of the results (measurements).**
- **Editing (only in the case of blunders or transcription errors).**
- **Analysis.**
- **Reporting**

Common mistakes:

- **Simplification: arithmetic mean is inside specifications while 50% of test results can be below specified minimum.**
- **Alteration of data: attempts can be made to change data to compensate for poor quality control and/or experimental design.**
- **Projection of data: test range or region is not adequately represented by the results.**

Data Collection

Interpretation:

Data interpretation is the ability to understand and to extract maximum information from a quantity of data that is being determined by measurement and presented in various forms e.g. a single value, groups of data, tables, histograms, charts etc.

Requirements:

- **The data collector must understand the data as thoroughly as possible in terms of:**
  - ✓ **the purpose for which the data source was selected or designed to address.**
  - ✓ **what, if any, surprises the data may have produced, and**
  - ✓ **what, if anything, such surprises may imply**

- **The data must be reported to the client in a manner that is as clear, complete, and intuitively compelling as possible.**

Preparation of data

- Data should be arranged and presented in a systematic or organized format, so that some useful deductions for the population under consideration can be drawn from the data set.

- This is important as it is usual practice to base decisions on the quality of a population or a lot on the outcomes of tests on relatively small numbers of samples of the lot or population.

Data Quality

There is no such thing as a Perfect Measurement

Measurement Category:

- **Direct measurement:** **measurements that can actually be taken with gauges, instruments or by counting.**

  For example: height of a person (measured with a ruler)

- **Indirect measurement:** **a technique that uses proportions to find a measurement when direct measurement is not possible.**

  For example: the radiation received by a nuclear gauge is measured and will be an indication of the soil density.

Data Quality

Instrumentation Reliability:

Instrumentation Reliability is defined as the extent to which an instrument consistently measures what it is supposed to, i.e. The same results will be obtained again if the measurement is performed again.

Types of Reliability:

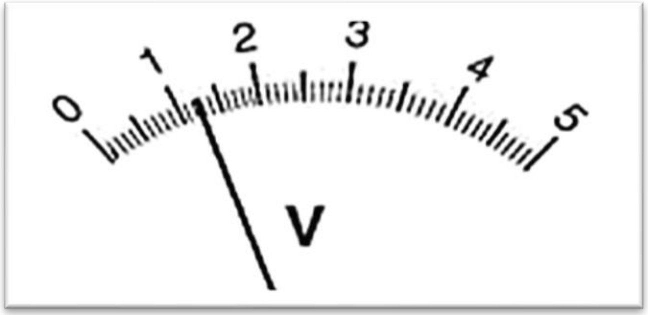
- **Internal consistency (Consistency of the items) is typically a measure based on the correlations between different items on the same test.**
- **Test-retest Reliability (Consistency over time). Test-retest is the variation in measurements taken by a single person or instrument on the same item and under the same conditions when tested one after the other.**
- **Interrater Reliability (Consistency between raters). Interrater Reliability describes the degree of agreement among raters.**

Data Quality

Analogue vs. Digital

Analogue:

When reading of an analogue display the precision of readings is limited by our ability to read them.



Digital:

Digital meters normally have sufficient digits to show values more precisely than it is possible to read an analogue display

Data Quality

Weighing Scales

Three types of weighing scales:

- **The spring scale: measures weight by the distance a spring deflects under its load.**

- **The strain gauge scale: an electronic version of a spring scale.**

- **The balance scale:**

Data Quality

Weighing Scales

Sources of error:

- Error in mass of reference weight.
- Air gusts, even small ones, which push the scale up or down.
- Settling airborne dust contributing to the weight.
- Calibration out of specification over time, due to drift in the circuit's accuracy, or temperature change.
- Condensation of atmospheric water on cold items.
- Evaporation of water from wet items.
- Vibration and seismic disturbances; for example, the rumbling from a passing truck.

Data Quality

Test Method:

A test method is a definitive procedure for the identification, measurement, and evaluation of one or more qualities, characteristics, or properties of a material, product, system or service that produces a test result.

A test method must specify :

- **the control over such factors as the test equipment.**
- **the test environment.**
- **the qualifications of the operator.**
- **the preparation of test specimens.**
- **the operating procedure for using the equipment in the test environment.**
- **the number of test specimens required.**
- **how measurements on the test specimens are to be combined to provide a test result.**

Data Quality

A quantitative test method may have three distinct stages:

- **The direct measurement or observation of dimensions or properties.**

  An observation or observed value (direct reading or interpolation) should be interpreted as the most elemental single reading or corrected reading obtained in the process of making a measurement.

- **The arithmetical combination of the observed values to obtain a single determination.**

  A test determination may be described as the process of calculating a property of a single test specimen from one or more observations.

- **The arithmetical combination of a number of determinations to obtain the test result of the test method.**

  A test result is the value obtained by carrying out the complete protocol of the test method once, being either a single test determination or a specified combination of a number of test determinations.

Data Quality

Test method types

Direct Test Method:

Test method where the results are directly related to the property of the materials, i.e. Sieve analysis

Indirect Test Method:

Test method where the test results are converted to the property of the material, i.e. when using a Schmidt Hammer, the rebound of the plunger is measured and converted to strength according to a certain empirical relationship

Data Quality

Test method types

Basic:

The test results are determined strictly in accordance with a definition of a value i.e. mass per volume (kg/m^3) or the compression strength of concrete by testing a concrete cube which reports force per unit area (kN/mm^2).

Rational:

Test results of this nature are expressed as a ratio where the basis of reference is a predefined standard, i.e. the degree of compaction of a layer; it is a ratio of the layer density expressed as a percentage of the MDD which is the basis of reference (95% of 100%in labs).

Conditional:

Test results which depend on certain conditions, i.e. the test result density depends on factors such as gradation, moisture content, etc. and is therefore conditional.

Data Quality

Test method types

Relational:

The property the test actually measured is related to another property which will be the test result of interest, i.e. the value of a bitumen spray rate is related to the mean of all the measurements of the least chip dimension (in millimetres) of a sample and the test result of interest (spray rate) is in terms of litres per square metre - not in millimetres anymore.

Interpolative:

The test result is obtained from a function and is not directly obtained from the test carried out, i.e. the MDD value is a good example of an interpolated test value. A moisture content versus dry density curve is constructed and from this relationship the Optimum Moisture Content is the moisture content where the relationship is maximum. The point at maximum density is called the Maximum Dry Density.

Data Quality

Test method types

Multiple:

Test values can be of more than one type, for example the CBR value at a specified compaction is rational, conditional, and interpolative

Data Quality

Test result reliability

Reliability:

- **Reliability refers to the consistency of a measure.**
- **A test is considered reliable if we get the same result repeatedly.**
- **The precision and accuracy of measurements are often collectively referred to as reliability.**
- **Reliability of data is a complex subject and two of the major elements are inherent, variation and human related errors.**
- **Variability is the result of changes in the conditions under which observations are made.**

Data Quality

Test result reliability

Variability regarding road building materials and construction test results is defined formally as follows:

$$E_t = E_m + E_i + E_s + E_g + E_e + E_o + E_r$$

Where:

E_t = total variation

E_m = variation due to inherent material variability.

E_i = Induced variation

E_s = variation due to sampling.

E_g = variation due to test method.

E_e = variation due to testing environment.

E_o = variation due to test operator.

E_r = random error.

Data Quality

Test result reliability

Total variation:

- **In most cases with unsystematic statistical comparison or reproducibility studies the variation obtained is actually the total variation.**
- **The actual degree or magnitude of the effect of the individual variation as mentioned above is normally unknown to the analyst.**
- **The degree of variation of a certain property does not have to be significant while the variation of another property can be significantly high.**
- **Thus the degree of interaction of the individual variation factors varies from situation to situation and must be determined every time a statistical analysis is carried out.**

- **For example, in a case of a comparison study carried out among testing organisations it is senseless to make a statement that one or more of the laboratories differ significantly when the magnitude of the material variability is not known**

Data Quality

Test result reliability

Variability due to the material:

- **The material variation is subdivided into components such as source and sample variation.**
- **Material variability is usually the major source of variation if not taken properly care of.**

Induced variability:

Induced variability is caused by certain construction actions and preparation for testing.

For example:

- **In the case of coarsely graded material the gradation of the material in a layer after compaction does not match up with the material in a stockpile.**
- **The gradations of the material prepared for certain tests such as the MDD- and CBR-test are not the same as that of the layer it represents.**

Data Quality

Test result reliability

Sampling:

- Materials for road construction work are not perfectly uniform in terms of the properties to be measured.
- Hence, sampling variability is, in certain cases, the main source of variability among test results.

Test method:

- The execution of a test method involves an interpretation of the test method by a specific test operator.
- Experimental factors - known and unknown - that can change the outcome of a test method are potential sources of variability in test results (Variations in reduction of material, variation in test apparatus etc.).

Data Quality

Test result reliability

Test environment:

- **Certain properties of certain materials are sensitive to environmental factors such as temperature, humidity, atmospheric pressure.**
- **The test method must describe or define a tolerable environmental condition.**

Test operator:

- **Every effort must be made in preparing a standard test method to eliminate the possibility of serious differences in interpretation.**
- **The test operator must be able to interpret test data correctly. To interpret data correctly requires knowledge and skill.**

Random error:

- **Random error or unexplained variation is the input of "mother-nature" or the lack of identifying all the factors contributing to the final variation.**
- **In most cases the effect of random error is usually trivial.**

Data Quality

Proficiency Schemes

Proficiency Testing Programs (PTP) are statistical quality assurance programs that enable laboratories to assess their performance in conducting test methods within their own laboratories when their data are compared against other laboratories that participate in the same program.

- **In the design of any proficiency scheme some of the contributing factors to variation must be kept constant while the factor of interest is assigned as a variable.**
- **Normally, the factors E_m(material variation), E_i (induced variation) and E_s (sampling variation) are kept constant while E_g (test method variation), E_e (test environment variation), and E_o (operator variation) are variables.**
- **Before a proficiency program can be executed a repeatability study must be carried out first to establish whether E_m and E_s in particular are constant and what are the respective ranges of the variables.**

Data Quality

Proficiency Schemes

- In such a repeatability study the magnitude of E_r (random error) must also be determined.
- The reference value(s) must also be agreed beforehand.
- All the samples must be prepared according to a set of rules which addresses all the factors that contribute to variability.
- Any procedure within a test method open to personal interpretation must be addressed in a list of instructions.
- All the participating laboratories must follow the required test according to a step-styled program of actions.
- Valid certificates of all equipment, where a calibration is required, must be available.

Data Quality

Specification:

- To attain an acceptable level of quality of a product or construction, it is required to comply with the provisions prescribed in a specification.
- A specification is a set of conditional statements and rules prescribing the quality required.
- For every property quality specified there must be a test method available to verify it.

False precision:

- False precision arises when numerical data are presented in a manner that implies better precision than is actually the case.
- False precision commonly arises when high-precision and low-precision data are combined, and in conversion of units.

Data Quality

Precision and Accuracy

Precision:

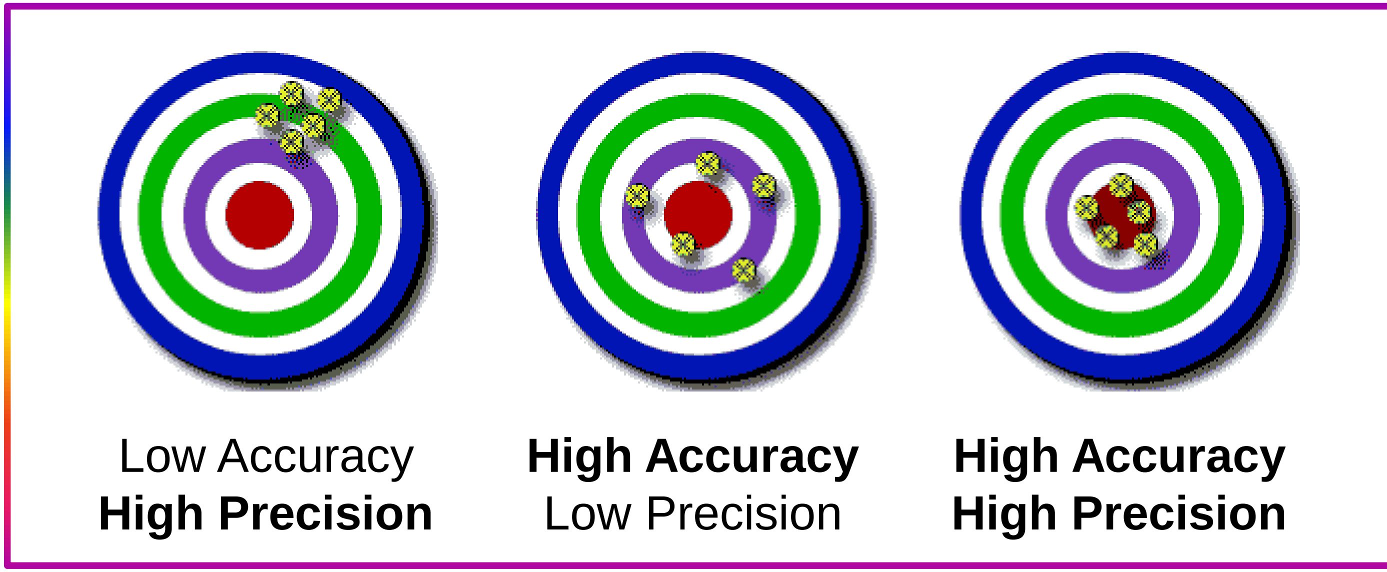
- **The precision of a measurement system, also called reproducibility or repeatability, is the degree to which repeated measurements under unchanged conditions show the same results.**
- **Precision is how close the measured values are to each other.**
- **Precision of an experiment/object/value is a measure of the reliability of the experiment.**

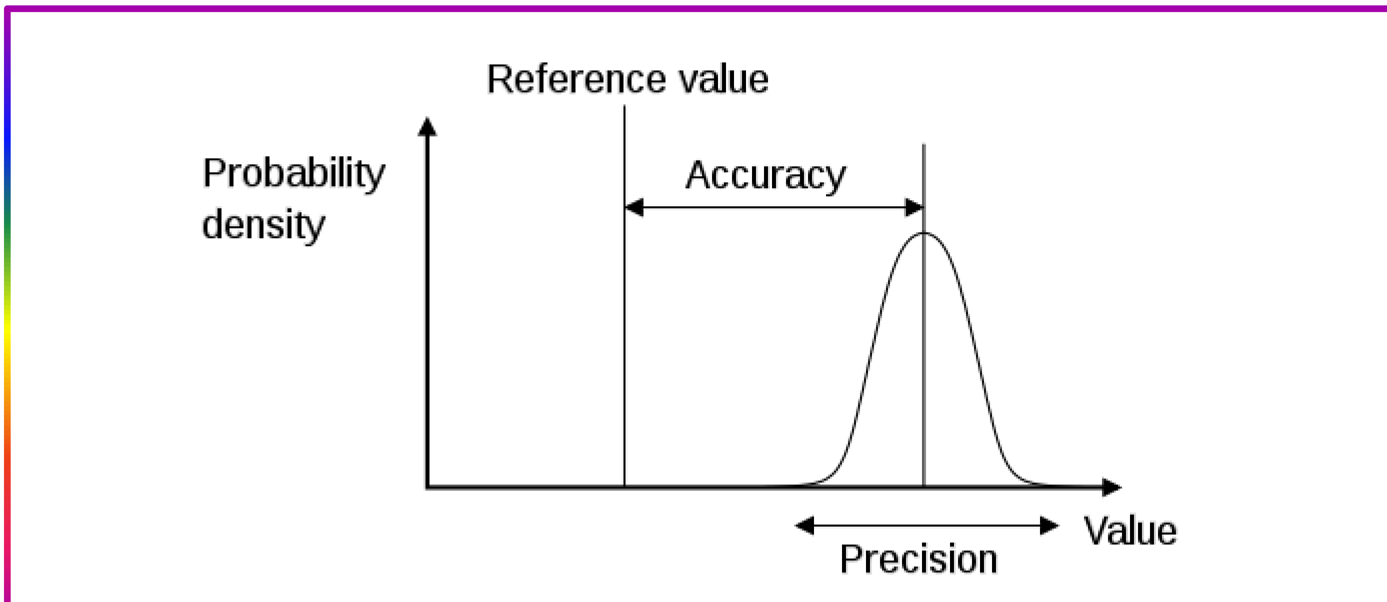
Accuracy:

- **Accuracy refers to the agreement between a measurement and the true or correct value.**
- **Accuracy is how close a measured value is to the actual (true) value.**
- **Accuracy refers to the closeness of a measured value to a standard or known value.**

Data Quality

Precision and Accuracy





Data Quality

Precision and Accuracy

- A measurement system can be accurate but not precise, precise but not accurate, neither, or both.
- If an experiment contains a systematic error, then increasing the sample size generally increases precision but does not improve accuracy.
- Eliminating the systematic error improves accuracy but does not change precision.

A measurement system is called valid if it is both accurate and precise.

Data Quality

Precision and Accuracy

Example:

The true mass of a gem is less than one millionth of a gram away from 5 grams, i.e. the mass of the gem is 4,999999 grams.

If the measurement is reported as 5 grams ± 0,000 003 grams, then the measurement has both high accuracy and high precision.

If it is reported as 5 grams ± 2 grams; then the measurement has high accuracy but low precision.

If it is reported as 8 grams ± 0,000 003 grams; then the measurement has high precision but low accuracy.

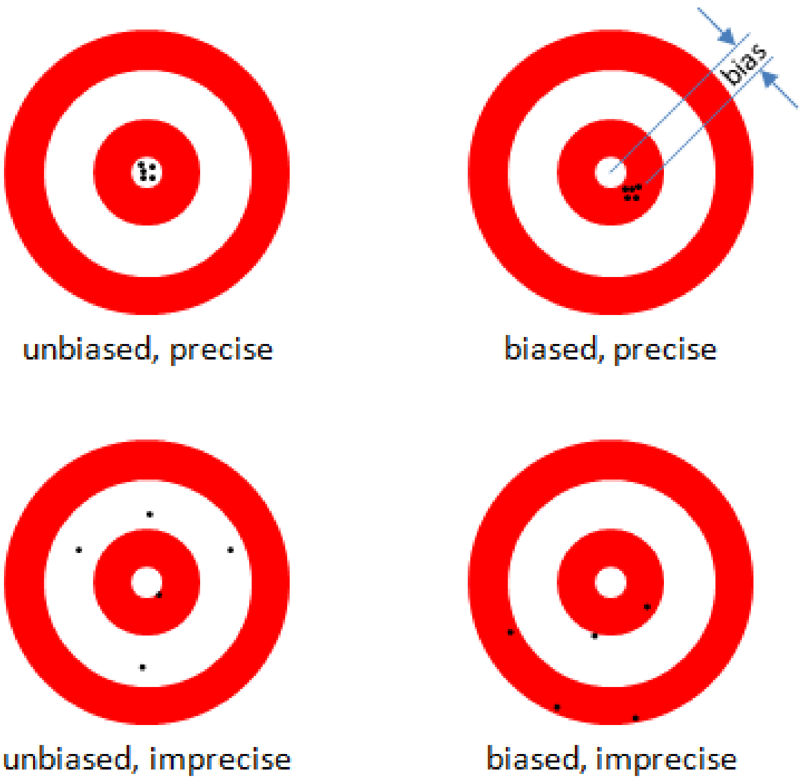
If it is reported as 8 grams ± 2 grams; then the measurement has both low precision and low accuracy.

Data Quality

Bias

Bias:

- **Bias is the consistent deviation of analytical results from the "true" value caused by systematic errors in a procedure.**
- **Bias is a systematic (built-in) error which makes all measurements wrong by a certain amount.**

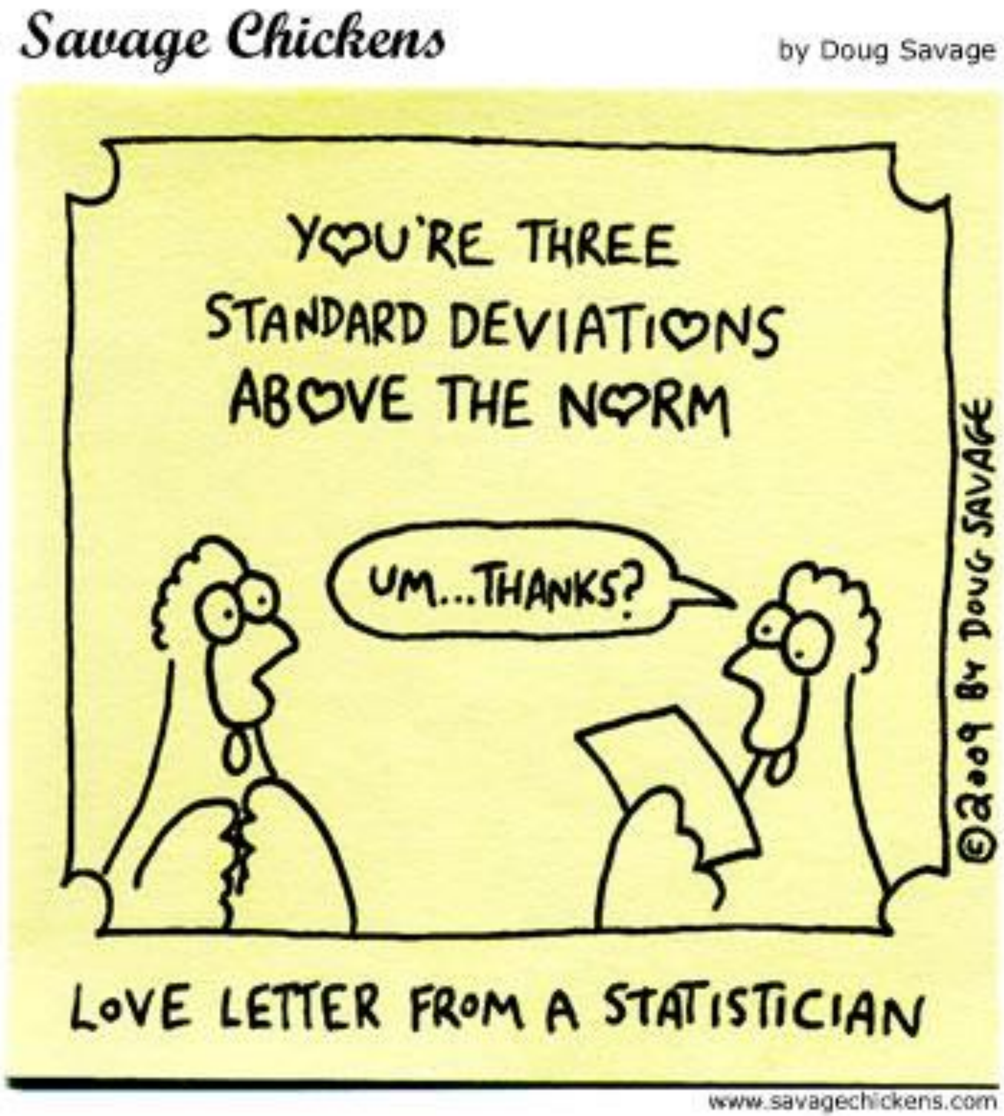


Data Quality

Bias

Examples:

- **The scales read "1 kg" when there is nothing on them**
- **You always measure your height wearing shoes with thick soles.**
- **A stopwatch that takes half a second to stop when clicked.**



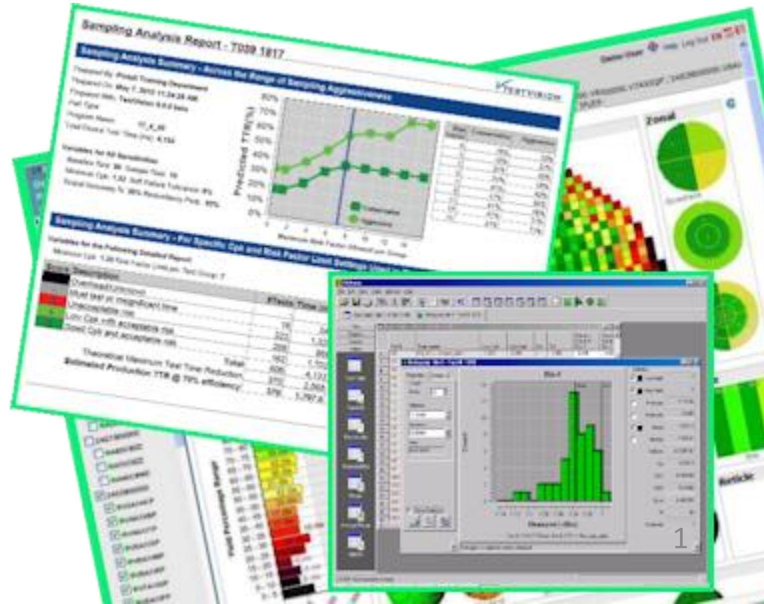
End of Chapter 3

Assessment and Analysis of Test Data

Presented by SARF

Presenter:
Ron Berkers

SARF
better roads



Module D
Judgement of Compliance

Quality Management Scheme:

- The information available on which to base a decision whether a product should be accepted or rejected is usually obtained from sampling a population.
- Even the highest standard of product sampling and testing cannot – and does not – provide flawless.
- There is always uncertainty about the correct decision due to sampling and testing inaccuracies and the inherent variation of the product being assessed.
- In judgement schemes no measurement is certain and consequently acceptance procedures are not always fair to both the supplier (contractor) and the client.
- Consequently statistical quality assurance procedures should be applied correctly and every precaution taken to minimise the risk of wrongful assessment of quality.

- An important aspect of the quality control is the detection of both random and systematic errors.
- For the detection of error type, as well as the quantification of the errors, statistical treatment of data is indispensable.
- In analytical work, the most important common operation is the comparison of data, or sets of data, to quantify accuracy (bias) and precision.
- The mere collection of facts will not constitute statistics unless they have been subjected to formal enquiry.
- Statistical methods should be used intelligently and carefully as their misuse will almost certainly lead to unsatisfactory and even hazardous outcomes.
- False conclusions will follow if data collected is incomplete and unreliable.

Judgement Schemes:

Judgment schemes can be divided into two classes:

Quality Control (QC)
Quality Assurance (QA).

Quality:

- Quality is determined by the users of product, clients or customers.
- If the specification does not reflect the true quality requirements, the product's quality cannot be guaranteed.

  For example, the parameters for a heating vessel should cover not only the material and dimensions but operating, environmental, safety, reliability and maintainability requirements.

- Managing quality on a project requires a clear understanding of the specific quality expectations of the customer followed by the implementation of a proactive plan to meet those expectations.
- The "proactive plan" contains a number of elements - the most important of which are the QC and QA activities that need to be performed.

Quality Control:

- Quality control (QC) is a procedure or set of procedures intended to ensure that a manufactured product or performed service adheres to a defined set of quality criteria or meets the requirements of the client or customer.
- Quality control is the more traditional way that businesses have used to manage quality.
- Quality control is concerned with checking and reviewing work that has been done.
- Quality control emphasises testing of products to uncover defects, and reporting to management who make the decision to allow or deny the release.

Quality Assurance:

- QA is defined as a procedure or set of procedures intended to ensure that a product or service under development (before work is complete, as opposed to afterwards) meets specified requirements.
- QA can be described as the systematic monitoring and evaluation of the various aspects of a project, service or facility to maximize the probability that minimum standards of quality are being attained by the production process.
- Two principles included in QA are: "Fit for purpose", the product should be suitable for the intended purpose; and "Right first time", mistakes should be eliminated.
- QA includes regulation of the quality of raw materials, assemblies, products and components, services related to production, and management, production and inspection processes.

Statistical Population:

A population is any entire collection of people, animals, plants or things from which we may collect data. It is the entire group we are interested in, which we wish to describe or draw conclusions about.

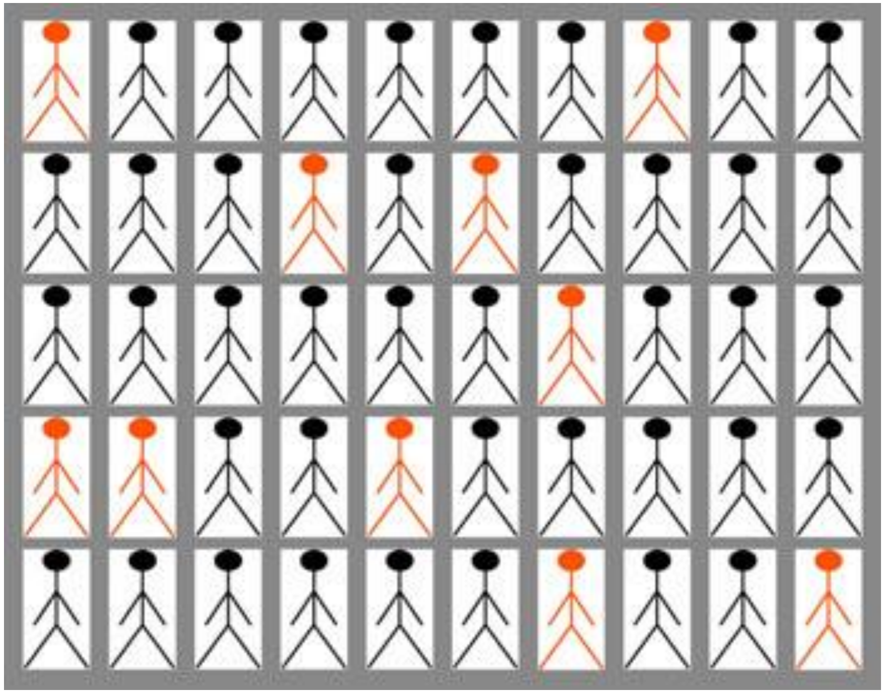
Sample:

- A sample is a group of units selected from a larger group (the population).
- By studying the sample it is hoped to draw valid conclusions about the larger group.
- A sample is generally selected for study because the population is too large to study in its entirety.
- The sample should be representative of the general population.

Sampling

Random Sample:

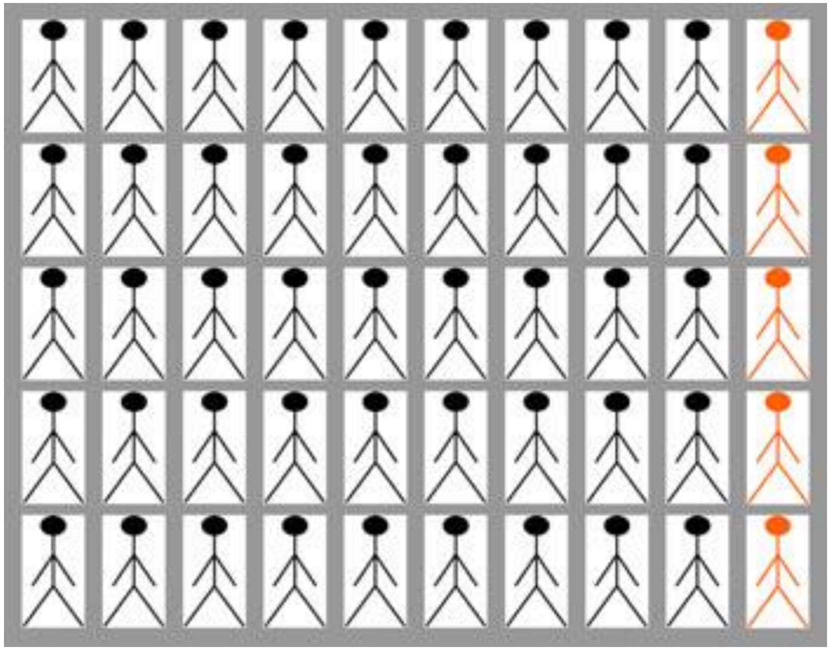
- Random sampling is the simplest method of probability sampling.
- Within a particular study population everyone has an equal chance of inclusion in the sample.
- It is considered ‘fair’ and therefore allows findings to be generalized to the whole population from which the sample was taken.



Sampling

Systematic Sample:

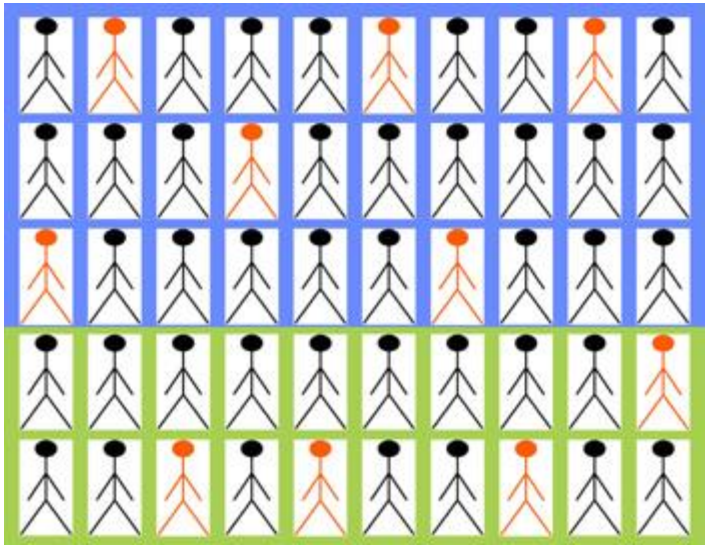
- In systematic sampling, individuals are chosen at *regular intervals* using a sampling frame to help you do this.
- Systematic sampling does not give every individual in a population the same chance of selection.
- It is usually easier and quicker to obtain a systematic sample than a random one.
- It is important to ensure that the sampling method does not introduce a consistent bias into the sample.



Sampling

Stratified Sample:

- Stratified random sampling involves dividing your population into various subgroups and then taking a simple random sample within each group.
- This will ensure that your sample represents key subgroups of the population.
- Representation of the subgroups can be proportionate or disproportionate.
- The basic purpose of stratification as compared to simple random sampling is to obtain a reduction in sampling error or, synonymously, an increase in precision.



Sampling

Click picture

Sample Size:

- How large should a sample be in any specific situation?
- If a sample is used which is larger than necessary, resources are wasted.
- If the sample is smaller than required, the risk of wrongful decisions may be increased and the objectives of the analysis not be achieved.
- Two questions must be answered to specify the required sample size:

  What degree of precision is desired?

  How probable must it be so that the desired precision will be obtained?
- The greater the degree of desired precision, the larger will be the necessary sample size.
- Also, the greater the probability specified for obtaining the desired precision, the larger will be required sample size.

Sample size *n*:

$$n = \left(\frac{z_\alpha \cdot \sigma}{\varepsilon}\right)^2$$

Where:

n	**is the number of observations or measurements required for a statistical sample aiming for a certain precision.**
σ	**is the population standard deviation.**
ε	**is the required precision aimed at.**
z_α	**is a standardized factor for a confidence level of α .**

For α = 5% it follows:

$$z_\alpha = z_{0,05} = \begin{cases} 1{,}65 \text{ if a one-sided case} \\ 1{,}96 \text{ if a two-sided case} \end{cases}$$

Example:

Determine the statistical sample size for an investigation to determine the layer compaction with a confidence interval of 95%. The precision error required should be equal or less than 0,5% . The assigned standard deviation is 1,8%.

Solution:

This is a one-sided case, because only a single limit for compaction, i.e. minimum, is specified.

$$z_{0,05} = 1,65$$

$$n = \left(\frac{(1,65) \cdot (1,8)}{0,5}\right)^2$$

$$n = 35$$

Example:

Determine the statistical sample size for determining Maximum Dry Density of a certain type of material. The precision error in estimating the mean density shall be less than 10 kg/m^3 with a confidence interval of 95%. The assigned standard deviation for the material is equal to 25 kg/m^3 .

Solution:

This is a two-sided case, because density and water content are specified.

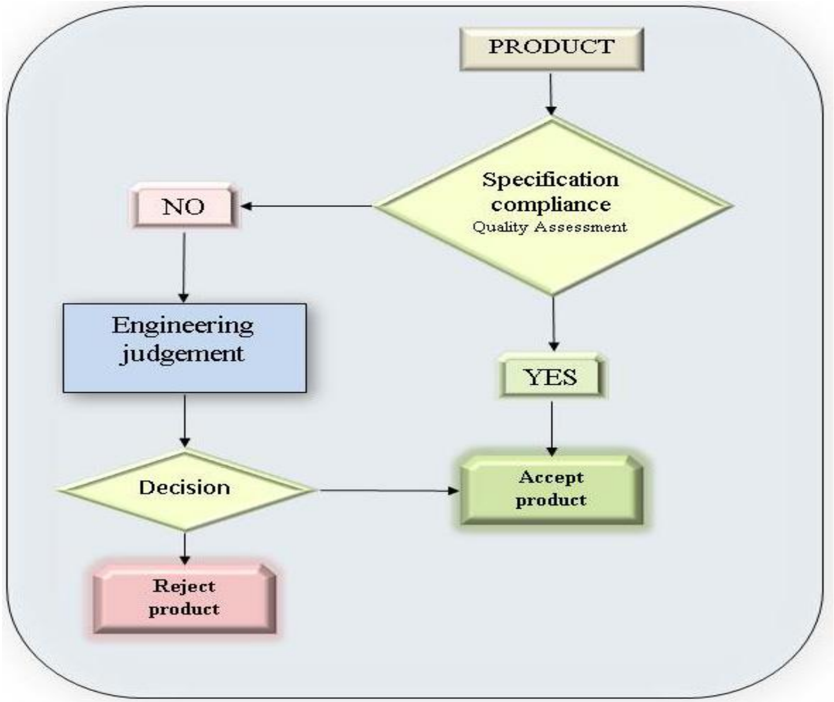
$$z_{\alpha/2=0,025} = 1,96$$

$$n = \left(\frac{(1,96)\cdot(25)}{10}\right)^2$$

$$n = 24$$

A Quality Assessment Model

- The information for a decision whether a property (product) is to be accepted as complying with a specific requirement or, otherwise, rejected is inevitably based on tests on samples of a population.
- Since this system does not provide comprehensive information about the product, quality assessment models have been developed to manage the risk of wrongful actions based on the outcomes of limited information.
- The quality assessment model used in the road construction industry can be schematically pictured as follows:

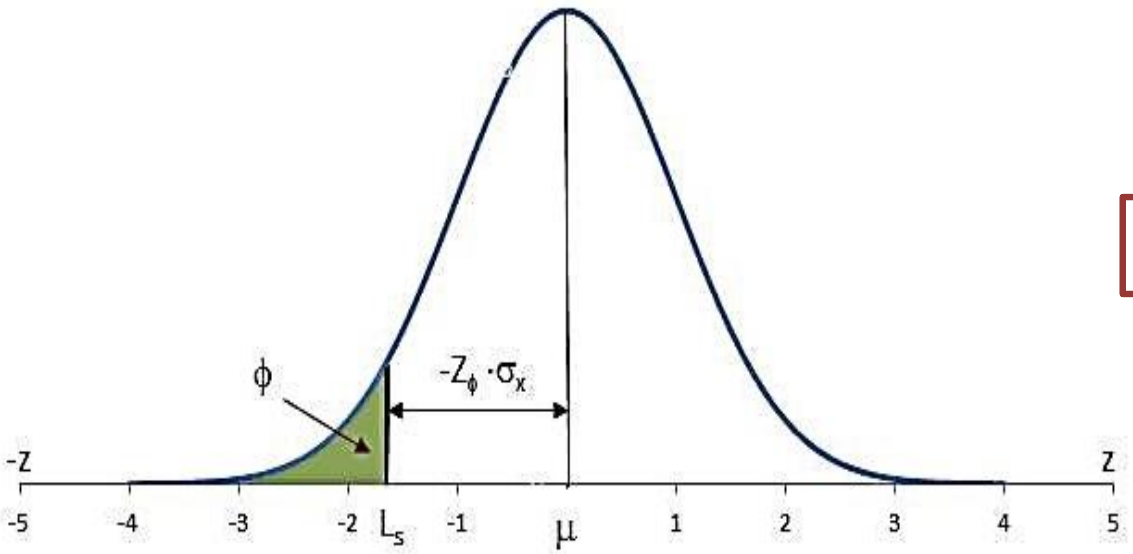


A Quality Assessment Model

- Almost all of the properties of road building materials which we assess for acceptance have near-normal distribution.
- The normal distribution is fully described by the two population parameters, i.e. the population mean (μ) and the population standard deviation (σ).
- The problem that we have is that these two parameters are most of time unknown and must be estimated from the sample statistics.

A Quality Assessment Model

For a normally distributed population where a single lower specification acceptance limit L_s is specified:



$$\mu = L_s + z_\varphi \cdot \sigma$$

Where:

φ (phi)	**is the percentage defectives allowed in a normally distributed population of a certain property.**
z_φ	**is the standardised constant for a normal distribution which is related to φ.**
L_s	**the lower specification limit for individual property test values or measurements.**
σ	**is the standard deviation of the population.**
μ	**is the mean of the population**

A Quality Assessment Model

Example:

By analysing a very large number of compaction values an analyst determined the following:

$$\mu = 99{,}5$$
$$\sigma = 1{,}00$$

What is the percentage of the total defectives (φ) allowed if the required limit for an acceptable compaction specification is L_S = 98,7?

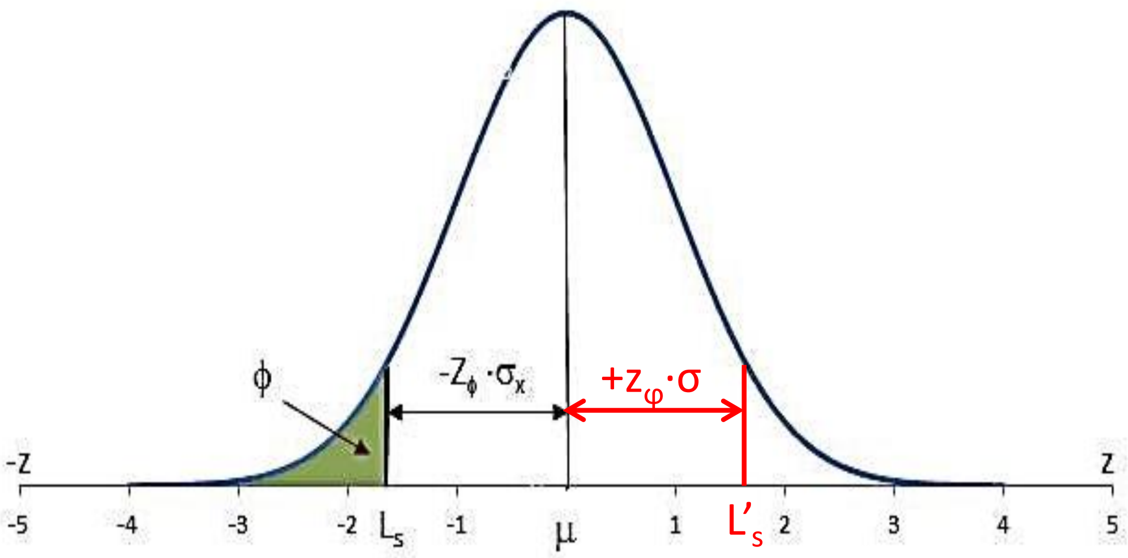
Solution:

$$\mu = L_s + z_\varphi \cdot \sigma \longrightarrow z_\varphi = \frac{\mu - L_s}{\sigma} = \frac{99{,}5 - 98{,}7}{1{,}00} = 0{,}800$$

Area φ = 0,5 – area z_φ = 0,5 – 0,288 = 0,212 (table 8)

Which is 21,2 %

A Quality Assessment Model



For a single upper limit specification is the acceptance limit L_s':

$$\mu = L_s' - z_\varphi \cdot \sigma$$

For dual specification limits the value of φ is normally relevant to both upper and lower specification limits and the following has to be applied simultaneously:

$$\mu = L_s + z_\varphi \cdot \sigma$$
$$\mu = L_s' - z_\varphi \cdot \sigma$$

Statistical Sample

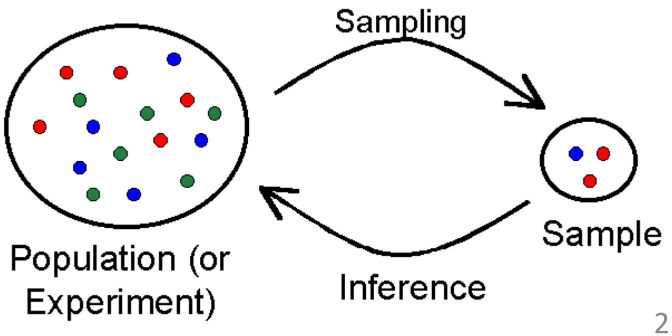
Statistical sample:

- Consists of two variables:

  sample mean, $\bar{x}$
  sample standard deviation, *s*

- and a constant, sample size *n.*

- as the sample size increases the variation in sample means and standard deviations decrease.



Acceptance Criterion

Terms used:

- L_A = lower acceptance limit for the sample mean
  L_R = the rejection limit for the sample mean.
  L_S = single lower acceptance limit
  αA = the risk when an acceptable product is rejected (contractor's risk).

- The risk to the contractor is called the α-risk:

  If the confidence interval is 95%, then the α-risk is 5% or 0.05.

  For example, there is a 5% chance that a part has been determined defective when it actually is not. One has observed or made a decision that a difference exists but there really is none.

- The risk to the client is called the β-risk:

  β-risk is the risk that the decision will be made that the part is not defective when it really is. In other words, when the decision is made that a difference does not exist when there actually is

Judgement Limits

For Compliance Limits:

Lower acceptance limit $L_A = L_s + F_A \cdot s$
Upper acceptance limit $L'_A = L_s - F_A \cdot s$
Lower rejection limit $L_R = L'_s + F_R \cdot s$
Upper rejection limit $L'_R = L'_s - F_R \cdot s$

F_A can be obtained from table 14(a) for single limit specification (α=5%) and table 14(c) for double limit specification (α=5%).

F_R can be obtained from table 14(b) for single limit specification (α=1%) and table 14(d) for double limit specification (α=1%).

Judgement Limits

Example:

The following sets of bitumen contents were obtained from a sections of asphalt, manufactured and laid according to a certain specification, given below:

Specification:

Bitumen content	5,50
Tolerance	0,25
Lower limit L_s	5,25
Upper limit L'_s	5,75
Φ - value	12%

Judgement Limits

Example (Continued):

Bitumen Content Test Results:

Lot	I
X_1	5,3
X_2	5,3
X_3	5,4
X_4	5,2
X_5	5,1

Statistics:

- $\bar{x}$ = sample mean = (5,3+5,3+5,4+5,2+5,1) / 5 = 5,26
- $s = \sqrt{}((5,3-5,26)^2+(5,3-5,26)^2+(5,4-5,26)^2+(I5,2-5,26I)^2+(I5,1-5,26I)^2)/5-1$

  $= \sqrt{}(0,04^2+0,04^2+0,14^2+0,06^2+0,16^2) / 4$

  = 0,114
- n = 5
- F_A value (α=0,05;n=5;ϕ=12%) = 0,294 (out of table 14(c))
- F_R value (α=0,01;n=5;ϕ=12%) = 0,024 (out of table 14(d))

Judgement Limits

Example (Continued):

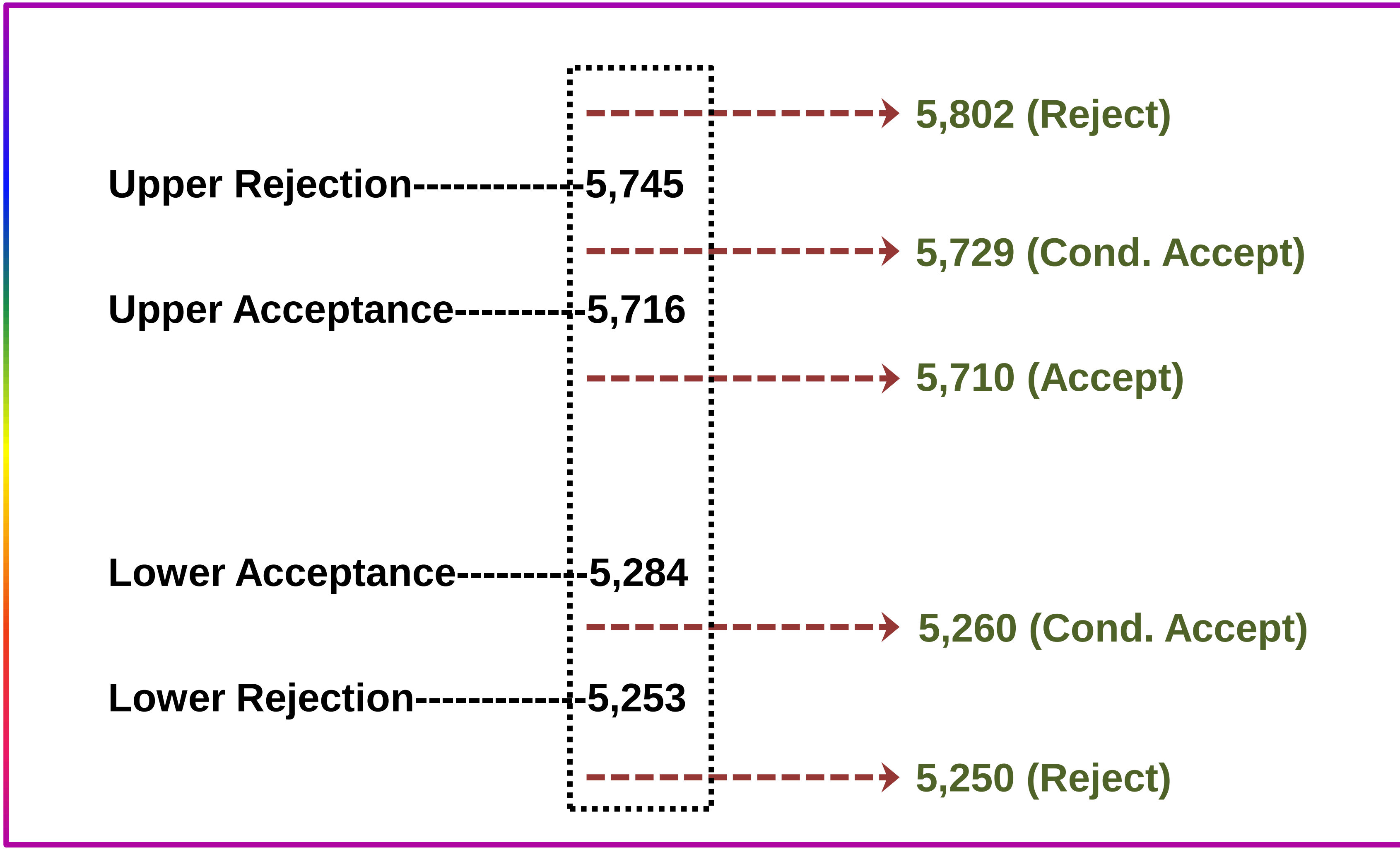
$\bar{x}$	5,260
s	0,114
n	5
F_A value	0,294
F_R value	0,024

Compliance Limits:

- Lower acceptance limit $L_A = L_s + F_A \cdot s$
  $= 5{,}25+(0{,}294 \times 0{,}114)$
  $= 5{,}284$
- Upper acceptance limit $L'_A = L_s – F_A \cdot s$
  $= 5{,}75 – (0{,}294 \times 0{,}114)$
  $= 5{,}716$
- Lower rejection limit $L_R = L'_s + F_R \cdot s$
  $= 5{,}25 + (0{,}024 \times 0{,}114)$
  $= 5{,}253$
- Upper rejection limit $L'_R = L'_s – F_R \cdot s$
  $= 5{,}75 – (0{,}024 \times 0{,}114)$
  $= 5{,}745$

- The test results are accepted conditionally (mean = 5,260 lies in transition zone ; 5,710 = Accept ; 5,250 = Reject).

Judgement Limits



- **The test results are accepted conditionally (mean = 5,260 lies in transition zone ; 5,710 = Accept ; 5,250 = Reject; 5,729 = transition zone; 5,802 = Reject).**

Resubmission of lots

In order to establish analytically whether there is a significant difference between two sets of values, the following procedure is normally followed:

1. The variance of the two samples is compared for significant dissimilarity.
2. Where the difference in variance is deemed to be insignificant, i.e. occurring by chance, the means of the sample sets will be compared.

Variance:

1. F-value is calculated with help of : $F = \frac{S_1^2}{S_2^2}$

   Where

   s_1 is the greater value of the two sample standard deviations
   s_2 is the lesser valued of the two sample standard deviations

2. The value of $F_{0,05\ v1;v2}$ is obtained from Table 10.

If $F > F_{0,05\ v1;v2}$ a significant difference is deemed to occur between the two sets of test data.
If $F < F_{0,05\ v1;v2}$ the difference is not necessarily significant and requires that a t-test be conducted.

Resubmission of lots

Means:

1. The t-test is conducted as follows:

$$s = \sqrt{\frac{(n_a - 1)s_a^2 + (n_b - 1)s_b^2}{n_a + n_b - 2}}$$

Where the suffixes a and b refer to the two sets of data being considered.

2. t is calculated as follows:

$$t = |\overline{x}_a - \overline{x}_b| / s \cdot \sqrt{1/n_a + 1/n_b}$$

3. The value of $t_{0,05;v}$ is obtained from Table 9 where

$$v = n_a + n_b - 2$$

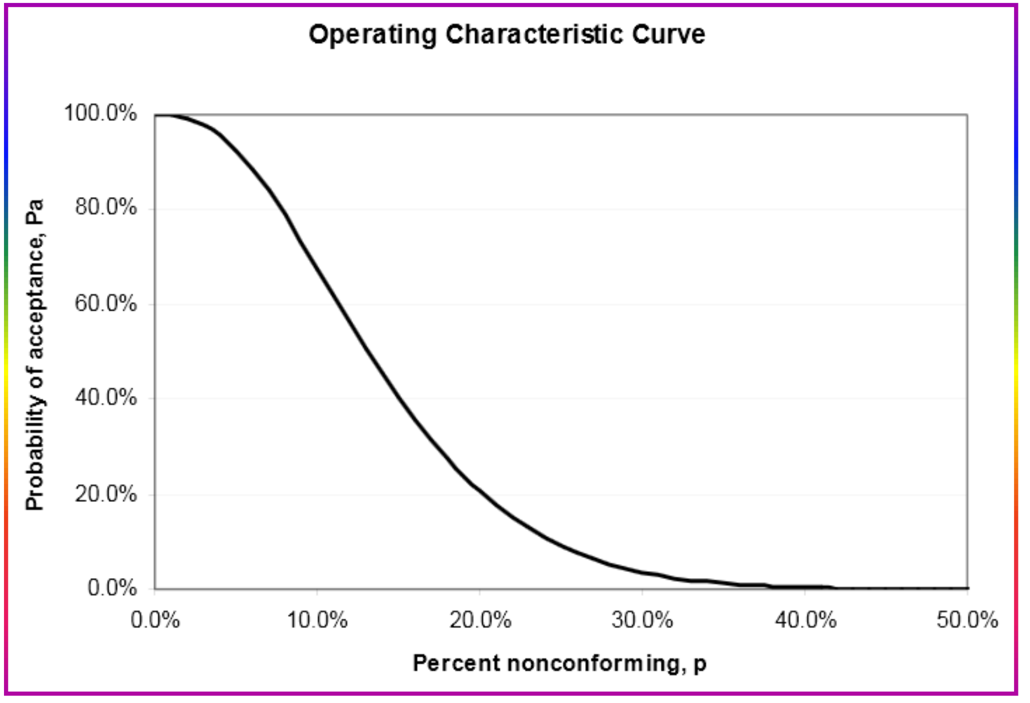
Where $t > t_{0,05;v}$ a significant difference between the sets of results is indicated.
Where $t < t_{0,05;v}$ the difference in means is not significant.

The Operating Characteristic Curve

The OC Curve:

The Operating Characteristic (OC) curve describes the probability of accepting a lot as a function of the lot's quality.

The figure below shows a typical OC Curve:



As the lot percent nonconforming increases, the probability of acceptance decreases.

Process Control Charts

- The variability in key quality characteristics usually arises from three sources:

1. improperly adjusted machines or equipment;
2. operator errors; and
3. defective raw materials

- The above sources of variability are not part of the chance cause pattern and are referred to as assignable causes.

- A major objective of statistical process control is to quickly detect the occurrence of assignable causes so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured.

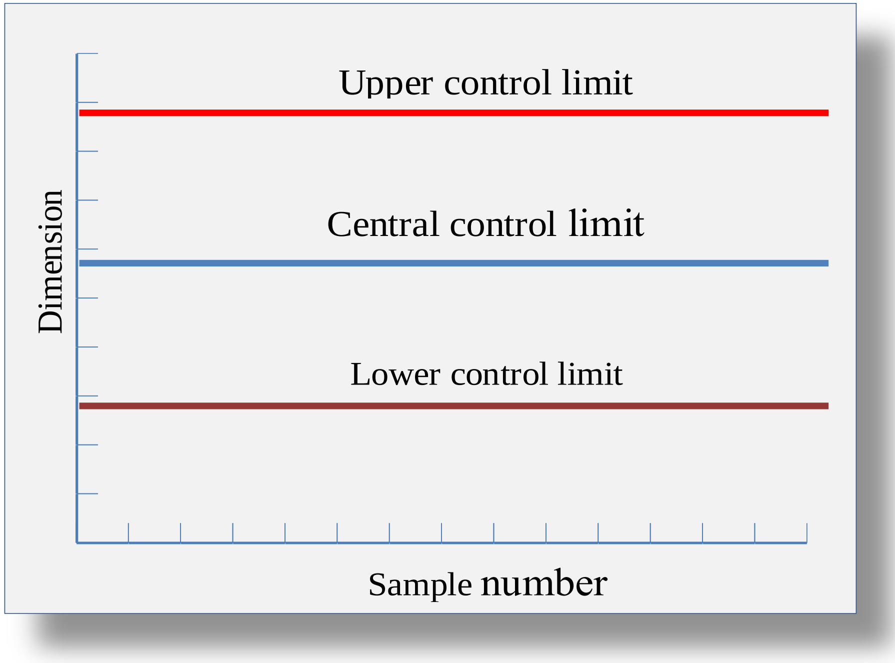
- The control chart is an online process-monitoring technique widely used for this purpose.

- The control chart can also provide information that is useful in improving the process.

Process Control Charts

The process control chart:

- A typical control chart is shown below:

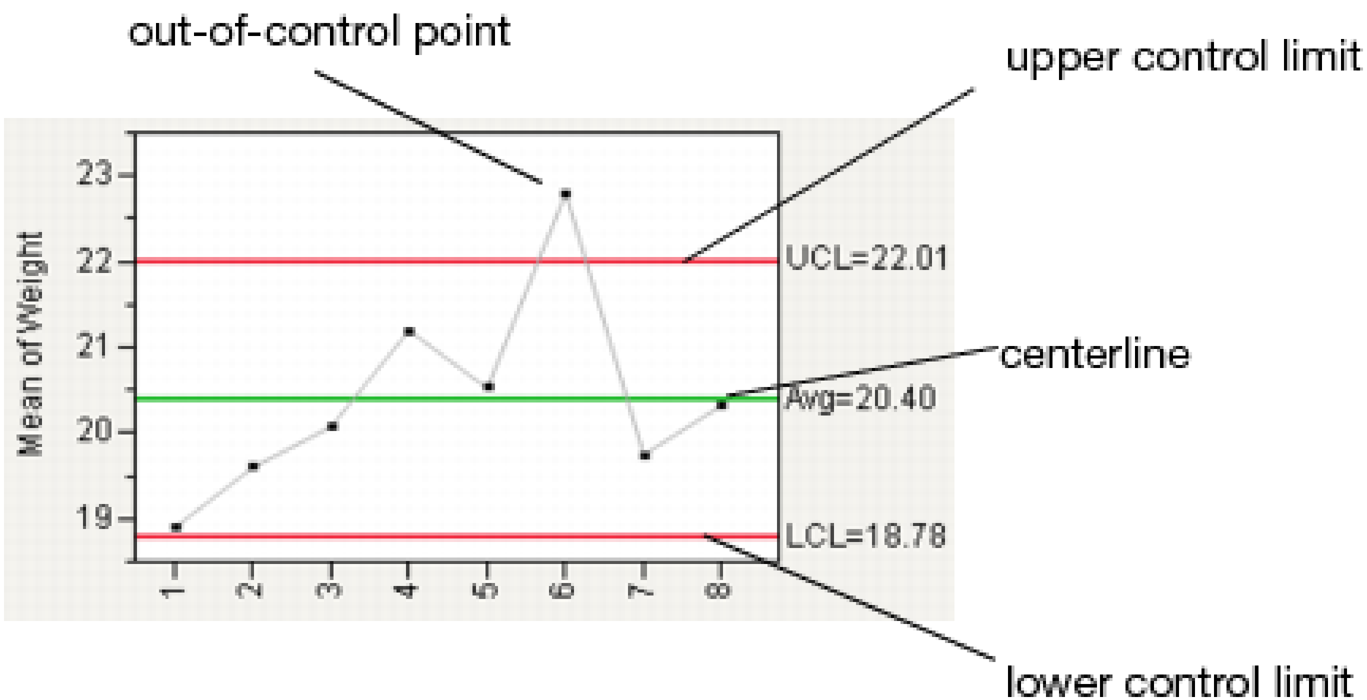


- The control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time.

Process Control Charts

- The chart contains a center line (CL) that represents the average value of the quality characteristic.

- Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL), are chosen so that if the process is in control (no assignable causes, only chance cause variations), nearly all of the sample points will fall between them.



$\overline{X}$ - Chart:

- For a known μ (population mean) and σ (standard deviation):

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$CL = \mu$$

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

- If μ and σ are not known (see table next page):

$$UCL = \overline{X} + A_2\overline{R}$$

$$CL = \overline{X}$$

$$LCL = \overline{X} - A_2\overline{R}$$

Process Control Charts

Table 1: Control Chart Constants

n	A_2	D_3	D_4
2	1.880	0	3.267
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.283	1.717
13	0.249	0.307	1.693
14	0.235	0.328	1.672
15	0.223	0.347	1.653

Process Control Charts

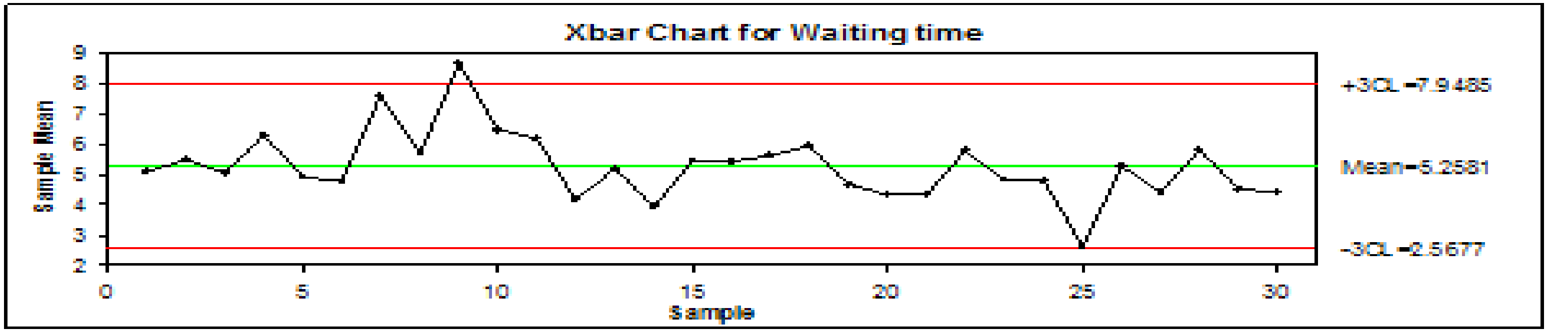
R – Chart:

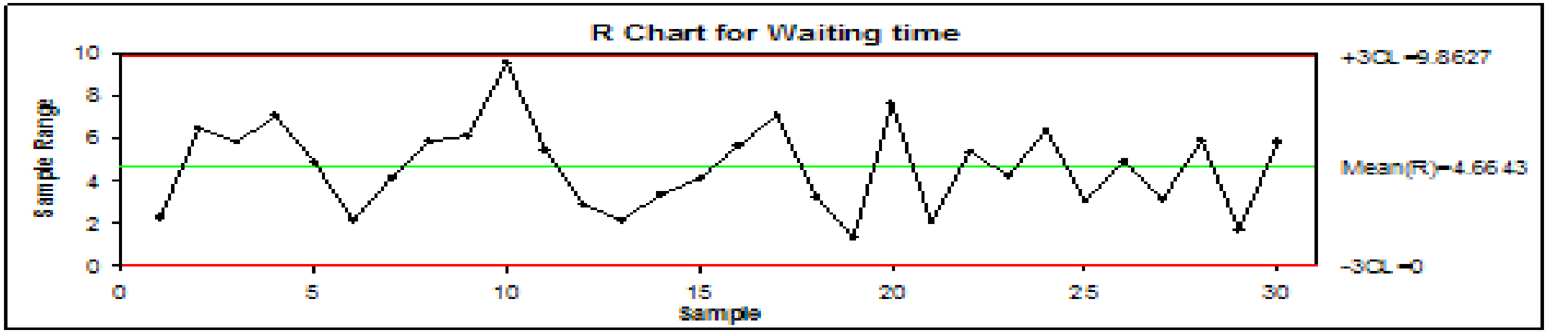
The center line and upper and lower control limits for R chart are:

$$UCL = D_4\overline{R}$$

$$CL = \overline{R}$$

$$LCL = D_3\overline{R}$$





Example:

A component part for a jet aircraft engine is manufactured by a casting process. The vane opening on this casting is an important functional parameter of the part.

The following table presents 20 samples of five results each:

n	X_1	X_2	X_3	X_4	X_5	Mean	Range
1	33	29	31	32	33	31,6	4
2	33	31	35	37	31	33,4	6
3	35	37	33	34	36	35,0	4
4	30	31	33	34	33	32,2	4
5	33	34	35	33	34	33,8	2
6	38	37	39	40	38	38,4	3
7	30	31	32	34	31	31,6	4
8	29	39	38	39	39	36,8	10
9	28	33	35	36	43	35,0	15
10	38	33	32	35	32	34,0	6
11	28	30	28	32	31	29,8	4
12	31	35	35	35	34	34,0	4
13	27	32	34	35	37	33,0	10
14	33	33	35	37	36	34,8	4
15	35	37	32	35	39	35,6	7
16	33	33	27	31	30	30,8	6
17	35	34	34	30	32	33,0	5
18	32	33	30	30	33	31,6	3
19	25	27	34	27	28	28,2	9
20	35	35	36	33	30	33,8	6

Process Control Charts

Example (Continued):

Mean of means: $\bar{X} = 33{,}3$
Mean of ranges: $\bar{R} = 5{,}8$

Out of table 1 for n=5

$$A_2 = 0{,}577$$

For the $\bar{X}$ - Chart:

$$\begin{aligned} UCL &= \bar{X} + A_2\bar{R} \\ &= 33{,}3 + 0{,}577 \times 5{,}8 \\ &= 36{,}65 \end{aligned}$$

$$\begin{aligned} LCL &= \bar{X} - A_2\bar{R} \\ &= 33{,}3 - 0{,}577 \times 5{,}8 \\ &= 29{,}59 \end{aligned}$$

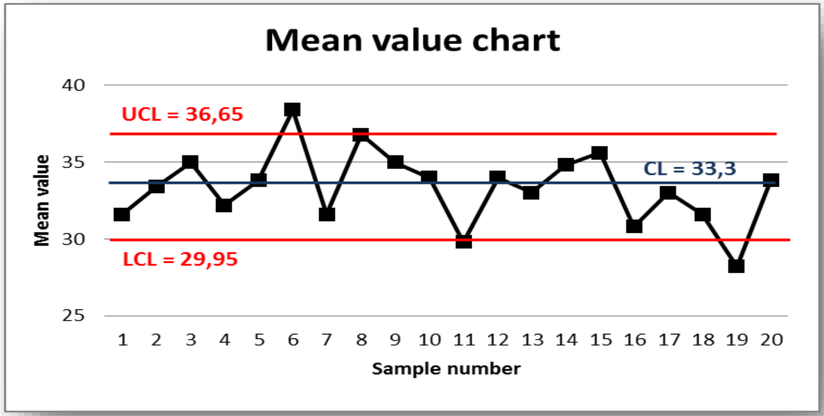
Process Control Charts

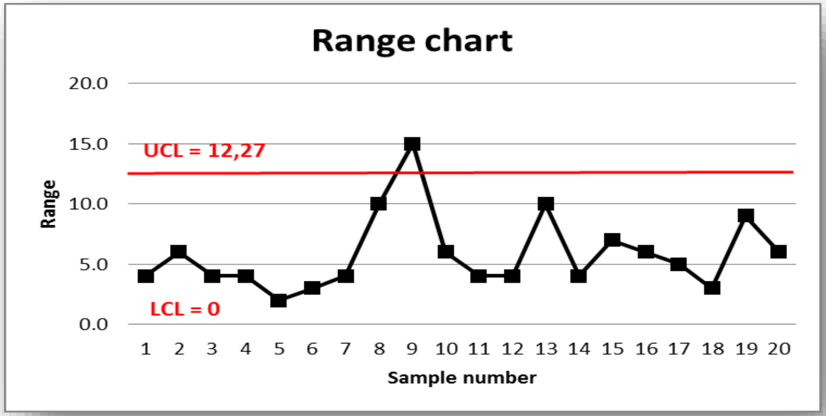
Example (Continued):

For the $\bar{R}$ - Chart:

$$UCL = D_4\bar{R} = 2{,}114 \times 5{,}8 = 12{,}27$$

$$LCL = D_3\bar{R} = 0 \times 5{,}8 = 0$$



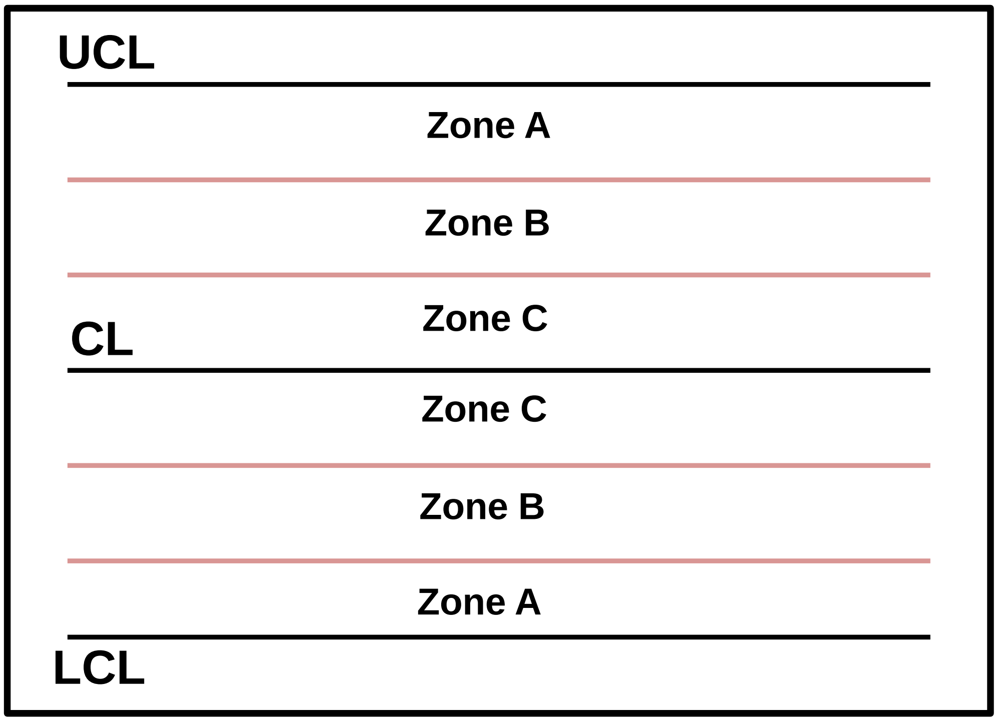


Samples 6, 8, 11, and 19 are out of control on the $\bar{X}$ -chart and sample 9 is out of control on the R –chart.

Process Control Charts

Interpretation of $\overline{X}$ - and R – Charts:

When interpreting $\overline{X}$ - and R control Charts, each of the regions between the CL - UCL and CL - LCL are divided into three equal zones, corresponding to 1σ and 2σ intervals as shown below:



Process Control Charts

Rules:

1. **Whenever a single point falls outside the 3-sigma control limits, a lack of control is indicated. Since the probability of this happening is rather small, it is very likely not due to chance.**

2. **Whenever at least 2 out of 3 successive values fall on the same side of the centreline and more than 2-sigma units away from the centreline (in Zone A or beyond), a lack of control is indicated.**

3. **Whenever at least 4 out of 5 successive values fall on the same side of the centreline and more than one-sigma unit away from the centreline (in Zones A or B or beyond), a lack of control is indicated.**

4. **Whenever at least 8 successive values fall on the same side of the centreline, a lack of control is indicated.**

Thank You

Tables

Contents

Table 1 - THE SEVEN SI BASE UNITS

Quantity[1]	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

[1] Quantity means measurable attribute of phenomena or matter.

Table 2 - DERIVED SI UNITS APPROVED BY CGPM

Quantity	Unit	Symbol	Formula
Frequency	hertz	Hz	1/s
Force	newton	N	$kg.m/s^2$
Pressure	pascal	Pa	N/m^2
Stress	pascal	Pa	N/m^2
Energy	joule	J	N.m
Work	joule	J	N.m
Quantity of heat	joule	J	N.m
Power	watt	W	J/s
Radiant flux	watt	W	J/s
Quantity of electricity	coulomb	C	A.s
Electric charge	coulomb	C	A.s
Electric potential	volt	V	W/A
Potential difference	volt	V	W/A
Electromotive force	volt	V	W/A
Electric capacitance	farad	F	C/V
Electric resistance	ohm	Ω	V/A
Electric conductance	siemens	S	A/V
Magnetic flux	weber	Wb	V.s
Magnetic flux density	tesla	T	Wb/m^2
Inductance	henry	H	Wb/A
Temperature (Celsius)	degree	°C	K
Luminous flux	lumen	lm	cd.sr

Table 3 - LIST OF COMMON DERIVED UNITS

Quantity	Unit	Symbol
Acceleration	metre per second squared	m/s^2
Acceleration (angular)	radian per second squared	rad/s^2
Velocity (angular)	radian per second	rad/s
Area	square metre	m^2
Concentration	mole per cubic metre	mol/m^3
Density	kilogram per cubic metre	kg/m^3
Heat capacity	joule per kelvin	J/K
Heat flux density	warr per square metre	W/m^2
Magnetic field strength	ampere per metre	A/m
Moment of force	newton metre	N·m
Power density	watt per square metre	W/m^2
Radiance	watt per square metre steradian	$W/(m^2.sr)$
Specific heat capacity	joule per kilogram kelvin	J/(kg·K)
Specific energy	joule per kilogram kelvin	J/kg
Surface tension	newton per metre	N/m
Thermal conductivity	watt per metre kelvin	W/(m.K)
Velocity	metre per second	m/s
Viscosity (dynamic)	pascal second	Pa·s
Viscosity (kinematic)	square metre per second	m^2/s
Viscosity (Poiseuille)	kilogram per second per metre	$kg \cdot s^{-1}\ m^{-1}$
Volume	cubic metre	m^3

Table 4 - PREFIXES AND SYMBOLS TO INDICATE MAGNITUDE

Prefix	Symbol	Multiplication factor
yotta	Y	$1{,}0 \times 10^{24}$
zetta	Z	$1{,}0 \times 10^{21}$
exa	E	$1{,}0 \times 10^{18}$
peta	P	$1{,}0 \times 10^{15}$
tera	T	$1{,}0 \times 10^{12}$
giga	G	$1{,}0 \times 10^{9}$
mega	M	$1{,}0 \times 10^{6}$
kilo	k	$1{,}0 \times 10^{3}$
hecto*	h	$1{,}0 \times 10^{2}$
deka*	da	$1{,}0 \times 10^{1}$
deci*	d	$1{,}0 \times 10^{-1}$
centi*	c	$1{,}0 \times 10^{-2}$
milli	m	$1{,}0 \times 10^{-3}$
micro	□	$1{,}0 \times 10^{-6}$
nano	n	$1{,}0 \times 10^{-9}$
pico	p	$1{,}0 \times 10^{-12}$
femto	f	$1{,}0 \times 10^{-15}$
atto	a	$1{,}0 \times 10^{-18}$
zepto	z	$1{,}0 \times 10^{-21}$
yocto	y	$1{,}0 \times 10^{-24}$

* To be avoided where practical

Table 5 (a) - EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Acceleration

To convert from	to	Multiply by
ft/s^2	metre per second squared (m/s^2)	3,048 000*E-01
Free fall, standard (g)	metre per second squared (m/s^2)	9,806 650*E+00
in/s^2	metre per second squared (m/s^2)	2,540 000*E-02

Angle

To convert from	to	Multiply by
degree	radian (rad)	1,745 329 E-02
minute	radian (rad)	2,908 882 E-04
second	radian (rad)	4,848 137 E-06

Area

To convert from	to	Multiply by
acre	square metre (m^2)	4,046 873 E+03
ft^2	square metre (m^2)	9,290 304*E-02
hectare	square metre (m^2)	1,000 000*E+04
in^2	square metre (m^2)	6,451 600*E-04
mi^2 (international)	square metre (m^2)	2,589 988 E+06

Torque (bending moment)

To convert from	to	Multiply by
dyne·cm	newton metre (N·m)	1,000 000*E-07
kgf·m	newton metre (N·m)	9,806 650*E+00
lbf·in	newton metre (N·m)	1,129 848 E-01
lbf·ft	newton metre (N·m)	1,355 818 E+00

Electricity and magnetism

To convert from	to	Multiply by
ampere hour	coulomb (C)	3,600 000*E+09
faraday (physical)	coulomb (C)	9,652 19 E+04
gauss	tesla (T)	1,000 000*E-04
gilbert	ampere (A)	7,957 747 E-01
maxwell	weber (Wb)	1,000 000*E-08
mho	siemens (S)	1,000 000*E+00
oersted	ampere per metre (A/m)	7,957 747 E+01

Table 5 (b) - EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Energy

To convert from	to	Multiply by
British thermal unit (international)	joule (J)	1,055 056 E+03
calorie (international)	joule (J)	4,186 800*E+00
electronvolt	joule (J)	1,602 19 E-19
erg	joule (J)	1,000 000*E-07
ft·lbf	joule (J)	1,355 818 E+00
kW·h	joule (J)	3,600 000*E+06
ton (nuclear equivalent of TNT)	joule (J)	4,184 E+09

Energy per unit area time

To convert from	to	Multiply by
Btu (international)/(ft^2·s)	watt per metre (W/m^2)	1,135 653 E+04
cal (thermochemical)/(cm^2·s)	watt per metre (W/m^2)	4,184 000*E+04
erg/(cm^2·s)	watt per metre (W/m^2)	1,000 000*E-03
W/cm^2	watt per metre (W/m^2)	1,000 000*E+04
W/in^2	watt per metre (W/m^2)	1,550 003 E+03

Force

To convert from	to	Multiply by
dyne	newton (N)	1,000 000*E-05
kilogram-force	newton (N)	9,806 650*E+00
kip (1000 ibf)	newton (N)	4,448 222 E+03
pound-force (lbf)	newton (N)	4,448 222 E+00
poundal	newton (N)	1,382 550 E-01
ton-force	newton (N)	8,896 443 E+03

Heat

To convert from	to	Multiply by
Btu (international)·ft/(h·ft^2·°F) *thermal conductivity*	watt per metre kelvin [W/(m·K)]	1,730 735 E+00
Btu (international)·in/(h·ft^2·°F) *thermal conductivity*	watt per metre kelvin [W/(m·K)]	1,442 279 E-01
Btu (international)/ft^2	joule per square metre (J/m^2)	1,135 653 E+04
Btu (international)/(h·ft^2·°F) *thermal conductance*	watt per square metre kelvin [W/(m^2·K)]	5,678 263 E+00
cal (thermochemical)/cm^2	joule per square metre (J/m^2)	4,184 000*E+04
cal (thermochemical)/(cm^2·min)	watt per square metre (W/m^2)	6,973 333 E+02
cal (international)/g	joule per kilogram (J/kg)	4,186 800*E+03
cal (international)/(g·°C)	joule per kilogram kelvin [J/(kg·K)]	4,186 800*E+03

Table 5 (c) - EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Length

To convert from	to	Multiply by
angstrom	metre (m)	1,000 000*E-10
astronomical unit	metre (m)	1,495 979 E+11
chain (66 ft)	metre (m)	2,011 684 E+01
fathom (6 ft)	metre (m)	1,828 804 E+00
foot (international)	metre (m)	3,048 000*E-01
foot (Cape)	metre (m)	3,314 858 E-01
inch	metre (m)	2,540 000*E-02
light year	metre (m)	9,460 55 E+15
micron	metre (m)	1,000 000*E-06
mil	metre (m)	2,540 000*E-05
mile (international - nautical)	metre (m)	1,852 000*E+03
mile (international)	metre (m)	1,609 344*E+03
parsec	metre (m)	3,085 678 E+16
rod (16.5 ft)	metre (m)	5,029 210 E+00
yard	metre (m)	9,144 000*E-01

Light

To convert from	to	Multiply by
cd/in^2	candela per square metre (cd/m^2)	1,550 003 E+03
footcandle	lux (lx)	1,076 391 E+01
lambert	candela per square metre (cd/m^2)	3,183 099 E+03
lm/ft^2	lumen per square metre (lm/m^2)	1,076 391 E+01

Mass

To convert from	to	Multiply by
carat (metric)	kilogram (kg)	2,000 000*E-04
grain	kilogram (kg)	6,479 891*E-05
gram	kilogram (kg)	1,000 000*E-03
ounce (oz, avoirdupois)	kilogram (kg)	2,834 952 E-02
ounce (troy)	kilogram (kg)	3,110 348 E-02
pound (lb, avoirdupois)	kilogram (kg)	4,535 924 E-01
ton (long, 2240 ib)	kilogram (kg)	1,016 047 E+03
ton (short, 2000 lb)	kilogram (kg)	9,071 847 E+02

Mass per unit area

To convert from	to	Multiply by
lb/ft^2	kilogram per square metre (kg/m^2)	4,882 428 E+00

Table 5 (d) - EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Mass per unit length

To convert from	to	Multiply by
lb/ft	kilogram per metre (kg/m)	1,488 164 E+00
lb/in	kilogram per metre (kg/m)	1,785 797 E+01

Mass per unit volume

To convert from	to	Multiply by
g/cm^3	kilogram per cubic metre (kg/m^3)	1,000 000*E+03
lb/ft^3	kilogram per cubic metre (kg/m^3)	1,601 846 E+01
lb/in^3	kilogram per cubic metre (kg/m^3)	2,767 990 E+04
ton (long)/yd^3	kilogram per cubic metre (kg/m^3)	1,328 939 E+03
ton (short)/yd^3	kilogram per cubic metre (kg/m^3)	1,186 553 E+03

Power

To convert from	to	Multiply by
Btu (international)	watt (W)	2,930 711 E-01
erg/s	watt (W)	1,000 000*E-07
ft·lbf/s	watt (W)	1,355 818 E+00
horsepower (550 ft·lbf/s)	watt (W)	7,456 999 E+02
horsepower (electric)	watt (W)	7,460 000*E+02

Pressure or stress

To convert from	to	Multiply by
atmosphere (standard)	pascal (Pa)	1,013 250*E+05
bar	pascal (Pa)	1,000 000*E+05
centimetre of mercury (0°C)	pascal (Pa)	1,333 22 E+03
centimetre of water (4°C)	pascal (Pa)	9,806 38 E+01
dyne/cm^2	pascal (Pa)	1,000 000*E-01
gf/cm^2	pascal (Pa)	9,806 650*E+01
inch of mercury (32°F)	pascal (Pa)	3,386 38 E+03
inch of mercury (60°F)	pascal (Pa)	3,376 85 E+03
kgf/cm^2	pascal (Pa)	9,806 650*E+04
kgf/m^2	pascal (Pa)	9,806 650*E+00
millibar	pascal (Pa)	1,000 000*E+02
lbf/ft^2	pascal (Pa)	4,788 026 E+01
lbf/in^2	pascal (Pa)	6,894 757 E+03
psi	pascal (Pa)	6,894 757 E+03

Table 5 (e) - EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Radiation units

To convert from	to	Multiply by
curie	becquerel (Bq)	3,700 000*E+10
roentgen	coulomb per kilogram (C/kg)	2,580 000*E-04

Temperature

To convert from	to	Multiply by
degree Celsius	kelvin (K)	$T_K = t_{°C} + 273,15$
degree Fahrenheit	degree Celsius (°C)	$t_{°C} = (t_{°F} - 32)/1,8$
degree Fahrenheit	kelvin (K)	$T_K = (t_{°F} + 459,67)/1,8$
kelvin	degree Celsius (°C)	$t_{°C} = T_K - 273,15$

Time

To convert from	to	Multiply by
day	second (s)	8,640 000*E+04
day (sidereal)	second (s)	8,616 409 E+04
hour	second (s)	3,600 000*E+03
minute	second (s)	6,000 000*E+01
year (365 days)	second (s)	3,153 600*E+07

Velocity

To convert from	to	Multiply by
ft/h	metre per second (m/s)	8,466 667 E-05
ft/min	metre per second (m/s)	5,080 000*E-03
ft/s	metre per second (m/s)	3,048 000*E-01
km/h	metre per second (m/s)	2,777 778 E-01
knot (international)	metre per second (m/s)	5,144 444 E-01
mi/h (international)	metre per second (m/s)	4,470 400*E-01
rpm (r/min)	radian per second (rad/s)	1,047 198 E-01

Viscosity

To convert from	to	Multiply by
centipoise (dynamic)	pascal second (Pa·s)	1,000 000*E-03
centistokes (kinematic)	square metre per second (m^2/s)	1,000 000*E-06
ft^2/s	square metre per second (m^2/s)	9,290 304*E-02
poise	pascal second (Pa·s)	1,000 000*E-01
stokes	square metre per second (m^2/s)	1,000 000*E-04

Table 5 (f) EQUIVALENT VALUES FOR MISCELLANEOUS UNITS OF MEASURE

Volume

To convert from	to	Multiply by
barrel (oil, 42 gal)	cubic metre (m^3)	1,589 873 E-01
ft^3	cubic metre (m^3)	2,831 685 E-02
gallon (U.K.)	cubic metre (m^3)	4,546 092 E-03
in^3	cubic metre (m^3)	1,638 706 E-05
litre	cubic metre (m^3)	1,000 000*E-03
ounce (U.K. fluid)	cubic metre (m^3)	2,841 306 E-05
pint (U.K. fluid)	cubic metre (m^3)	5,682 45 E-02
yd^3	cubic metre (m^3)	7,645 549 E-01

Volume per unit time

To convert from	to	Multiply by
ft^3/min	cubic metre per second (m^3/s)	4,719 474 E-04
ft^3/s	cubic metre per second (m^3/s)	2,831 685 E-02
in^3/min	cubic metre per second (m^3/s)	2,731 177 E-07

Table 6 (a) - ISRM[2] SYMBOLS RELATING TO SOIL AND ROCK MECHANICS

Symbol	Definition	Symbol	Definition
Space		G_m	mass specific gravity
Ω, ω	solid angle	G_s	specific gravity of solids
l	length	G_w	specific gravity of water
b	width	F	force
h	height	T	tangential force
h	depth	W	weight
r	radius	γ	unit weight
A	area	γ_d	dry unit weight
V	volume	γ_w	unit weight of water
t	time	γ'	buoyant unit weight
v	velocity	γ_s	unit of solids
ω	angular velocity	T	torque
g	gravitational acceleration	I	moment of inertia
Periodic		W	work
T	periodic time	W	energy
f	frequency	**Mechanics**	
ω	angular velocity	e	void ratio
λ	wave length	n	porosity
Statics and dynamics		w	water content
m	mass	S_r	degree of saturation
ρ	density (mass density)	p	pressure
		u	pore water pressure
		σ	normal stress
		σ_x, σ_y, σ_z	stress components in rectangular coordinates

[2] ISRM: International Society for Rock Mechanics

Table 6 (b) - ISRM SYMBOLS RELATING TO SOIL AND ROCK MECHANICS

Symbol	Definition	Symbol	Definition
$\sigma_1, \sigma_2, \sigma_3$	principal stresses	t_{rel}	relaxation time
S_1, S_2, S_3	applied stresses (and reactions)	T_s	surface tension
σ_h	horizontal stress	q	quantity rate of flow; rate of discharge
σ_v	vertical stress	Q	quantity of flow
τ	shear stress	FS	safety factor
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	shear stress components in rectangular coordinates	**Heat**	
ε	strain	T	temperature
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	strain components in rectangular coordinates	β	coefficient of volume expansion
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	shear strain components in rectangular coordinates	**Electricity**	
θ	volume strain	I	electric current
E	Young's modulus; modulus of elasticity	Q	electric charge
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	principal strains	C	capacitance
G	shear modulus; modulus of rigidity	L	self-induction
c	cohesion	R	resistance
ϕ_s	angle of friction between solid bodies	ρ	resistivity
ϕ	angle of shear resistance (angle of internal friction)	A - Z	Measurements, readings, etc. Avoid the letters {I,O}
h	hydraulic head	D	Maximum pore diameter
i	hydraulic gradient	ΔT	Difference in temperature

Table 6 (c) - ISRM SYMBOLS RELATING TO SOIL AND ROCK MECHANICS

Symbol	Definition	Symbol	Definition
j	seepage force per unit volume or seepage pressure per unit length	R	Modulus of rapture
k	coefficient of permeability	W	Breaking load
η	viscosity	t	Thickness
η_{pl}	plasticity (viscosity of Bingham body)	M_i	Mass of sample at condition *i*
t_{ret}	retardation time	M	Mass of wet sample
M_0	Mass of dry sample	L_s	Linear Shrinkage
C	Compressive strength	ρ_w	Density of water
G_t	Specific gravity of type *t*	γ_d	Dry unit mass of compacted sample
H_m	Resistance (wear) of type *m*	γ_m	Wet unit mass of compacted sample
σ	Flexural strength	γ_w	Unit mass of water
V_w	Volume of wet sample	γ_{max}	Maximum dry unit mass of compacted sample
V_d	Volume of dry sample	P	Percentage

Table 7 (a) - RANDOM NUMBERS

R_X	R_Y	R_X	R_Y	R_X	R_Y	R_X	R_Y
0,4556	0,8303	0,7061	0,7257	0,2877	0,5528	0,9627	0,4434
0,3926	0,3023	0,0074	0,9700	0,2077	0,5408	0,6333	0,8912
0,5805	0,4138	0,8389	0,4035	0,8455	0,6448	0,7733	0,6033
0,5835	0,3826	0,8163	0,3708	0,5930	0,4115	0,5748	0,5071
0,1066	0,6280	0,0762	0,2781	0,7822	0,1923	0,9405	0,0238
0,5006	0,2589	0,8118	0,9879	0,5018	0,6943	0,8313	0,0245
0,8898	0,6869	0,4323	0,3381	0,0381	0,6559	0,2388	0,9129
0,9266	0,3650	0,3488	0,7874	0,9418	0,6477	0,7835	0,8319
0,1704	0,6503	0,3841	0,1836	0,7193	0,9982	0,9368	0,9960
0,0913	0,7928	0,8078	0,6594	0,1491	0,8446	0,8691	0,1480
0,6989	0,0485	0,9966	0,6509	0,2232	0,3086	0,9222	0,1413
0,7966	0,9176	0,7195	0,0428	0,7134	0,7974	0,7071	0,6467
0,8601	0,5078	0,0477	0,4376	0,3629	0,4287	0,8268	0,9844
0,5418	0,0226	0,8902	0,5503	0,7027	0,5827	0,2235	0,0815
0,7990	0,3748	0,1532	0,7278	0,4928	0,5769	0,1517	0,5535
0,6485	0,9244	0,5263	0,5935	0,7888	0,4670	0,7756	0,9115
0,5451	0,3422	0,8918	0,8167	0,6332	0,9734	0,3920	0,8944
0,3856	0,5979	0,3599	0,0075	0,3718	0,5310	0,2776	0,9257
0,1377	0,0732	0,9298	0,7360	0,5953	0,4657	0,1620	0,6955
0,0941	0,8008	0,7734	0,6768	0,7055	0,2947	0,3252	0,8680
0,7510	0,8268	0,1467	0,8788	0,1065	0,1529	0,3201	0,9128
0,3124	0,6995	0,9244	0,1593	0,0219	0,3429	0,3207	0,0626
0,8188	0,0825	0,7600	0,6242	0,9352	0,0114	0,0940	0,3948
0,9012	0,4932	0,8711	0,3118	0,6569	0,9497	0,5982	0,0387
0,1597	0,1658	0,4497	0,9629	0,0157	0,5193	0,2052	0,5076
0,1682	0,0396	0,6971	0,9150	0,1747	0,7034	0,5484	0,1556
0,1022	0,8727	0,4840	0,1219	0,5733	0,2818	0,9948	0,7599
0,9937	0,4795	0,6362	0,2983	0,4931	0,1322	0,7434	0,2277
0,6094	0,0949	0,8439	0,1894	0,2495	0,6559	0,5476	0,4281
0,9551	0,8618	0,1977	0,9652	0,0082	0,3284	0,0623	0,1494
0,4043	0,4448	0,3482	0,7401	0,2266	0,3756	0,8177	0,1809
0,4521	0,7686	0,2914	0,2177	0,7200	0,5741	0,8158	0,5202
0,0944	0,8808	0,7787	0,4600	0,3531	0,1770	0,7536	0,7048
0,8316	0,9405	0,3523	0,7819	0,3566	0,9654	0,8708	0,2696
0,2974	0,3316	0,0051	0,7712	0,2684	0,4230	0,4224	0,3288
0,5129	0,9009	0,4660	0,4326	0,5000	0,0381	0,7764	0,2825
0,7654	0,3217	0,1099	0,4575	0,5280	0,7286	0,1360	0,6495
0,1124	0,5817	0,4932	0,6184	0,7104	0,3927	0,8879	0,0235
0,5105	0,5965	0,5136	0,2886	0,7283	0,5855	0,5748	0,1560
0,5690	0,9479	0,1953	0,0867	0,6518	0,3187	0,4927	0,1625

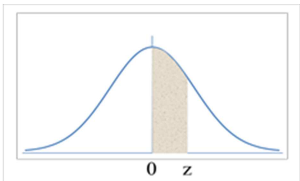
Table 7 (b) - RANDOM NUMBERS

R_X	R_Y	R_X	R_Y	R_X	R_Y	R_X	R_Y
0,3820	0,1007	0,5965	0,8991	0,8846	0,9585	0,0145	0,4074
0,8632	0,1386	0,2450	0,0455	0,0324	0,1641	0,2196	0,0171
0,2850	0,3431	0,5536	0,3574	0,3718	0,3556	0,9103	0,4660
0,4262	0,3039	0,9757	0,8067	0,9912	0,2563	0,9517	0,0534
0,7050	0,8165	0,9725	0,4663	0,3002	0,7502	0,3515	0,7757
0,0743	0,1984	0,0641	0,3583	0,4870	0,5112	0,3735	0,9859
0,0407	0,2307	0,0050	0,9261	0,1003	0,2567	0,7757	0,6796
0,8091	0,7243	0,0851	0,1323	0,7562	0,6265	0,1737	0,4048
0,5523	0,7115	0,5552	0,1812	0,9703	0,6869	0,5288	0,7967
0,8057	0,2622	0,1780	0,8668	0,1148	0,0595	0,7616	0,7384
0,9863	0,9256	0,9039	0,5450	0,5008	0,6750	0,4898	0,1458
0,0380	0,7963	0,6716	0,7317	0,5845	0,1522	0,8922	0,3778
0,2005	0,2058	0,3340	0,3251	0,3002	0,8022	0,6961	0,2715
0,9040	0,0391	0,7090	0,4537	0,5166	0,2565	0,2913	0,8021
0,7890	0,6760	0,7553	0,9485	0,6194	0,7221	0,9680	0,3686
0,8504	0,5571	0,8731	0,4411	0,2177	0,8590	0,2803	0,7033
0,7074	0,3758	0,3297	0,0860	0,9769	0,2855	0,5343	0,4074
0,9977	0,8947	0,8108	0,9086	0,5745	0,7061	0,4014	0,1110
0,8974	0,3863	0,0958	0,7776	0,7836	0,6657	0,6568	0,2585
0,7652	0,7003	0,8588	0,0028	0,6786	0,9288	0,0425	0,5181
0,9121	0,9543	0,5943	0,5577	0,9682	0,4830	0,2556	0,8179
0,4960	0,8506	0,6681	0,9269	0,4518	0,1681	0,0620	0,0052
0,5411	0,6176	0,4929	0,5795	0,6019	0,9301	0,5340	0,1321
0,0823	0,5759	0,8292	0,0657	0,2709	0,6997	0,4142	0,3656
0,4351	0,3301	0,2111	0,7405	0,5235	0,8968	0,6034	0,5228
0,5898	0,5849	0,4970	0,1105	0,5930	0,5589	0,7741	0,2308
0,7312	0,5867	0,5455	0,8073	0,9643	0,0954	0,1086	0,7123
0,8831	0,1899	0,0160	0,1845	0,5800	0,6661	0,1623	0,1951
0,6778	0,5484	0,2946	0,5408	0,1732	0,1853	0,8524	0,9481
0,2508	0,4317	0,5449	0,9675	0,7241	0,9510	0,5703	0,9406
0,2599	0,1670	0,8838	0,8209	0,0419	0,8983	0,4208	0,1282
0,0301	0,2047	0,6820	0,8206	0,4723	0,4785	0,5850	0,3624
0,8233	0,3106	0,9904	0,7716	0,7293	0,1633	0,8083	0,9261
0,2323	0,3817	0,0902	0,9120	0,8520	0,5731	0,0142	0,6934
0,7006	0,8880	0,1690	0,8860	0,2871	0,2316	0,0423	0,9154
0,1672	0,9389	0,1216	0,3413	0,0955	0,9441	0,5115	0,6295
0,8356	0,9744	0,4757	0,1516	0,2846	0,8017	0,8085	0,6952
0,0686	0,0813	0,4428	0,2647	0,2651	0,8103	0,2009	0,4543
0,4082	0,9355	0,0938	0,1748	0,4335	0,1443	0,0758	0,0153

Table 7 (c) - RANDOM NUMBERS

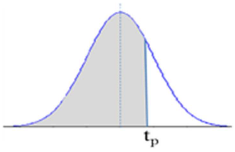
R_X	R_Y	R_X	R_Y	R_X	R_Y	R_X	R_Y
0,4600	0,5536	0,9587	0,3062	0,2134	0,2276	0,7213	0,9004
0,0756	0,8338	0,9438	0,2524	0,5331	0,2037	0,7566	0,5945
0,5186	0,1516	0,3822	0,7656	0,4961	0,8419	0,1553	0,7760
0,8928	0,1211	0,6541	0,0374	0,5316	0,8429	0,8498	0,5419
0,2231	0,7185	0,6788	0,7669	0,1713	0,7246	0,9464	0,5926
0,3500	0,5982	0,9655	0,0082	0,5067	0,4510	0,8388	0,8918
0,9490	0,0345	0,7893	0,6431	0,7710	0,6858	0,4462	0,9515
0,6752	0,4876	0,4918	0,4791	0,0462	0,6712	0,5764	0,7424
0,4329	0,7955	0,9068	0,9714	0,0950	0,7324	0,4146	0,2290
0,7701	0,9901	0,9114	0,5713	0,3180	0,4059	0,1361	0,5298
0,3980	0,7387	0,1013	0,6982	0,8356	0,1938	0,6449	0,7023
0,8354	0,4103	0,5922	0,3376	0,6950	0,9426	0,4368	0,1530
0,1784	0,7672	0,9448	0,2637	0,2026	0,8468	0,5882	0,6733
0,9953	0,2812	0,2163	0,8873	0,3804	0,0783	0,8685	0,0988
0,3452	0,0751	0,1360	0,7278	0,5484	0,0401	0,9792	0,5950
0,9232	0,4892	0,4057	0,5116	0,0320	0,2451	0,9166	0,5670
0,3301	0,2726	0,1259	0,8941	0,9933	0,8201	0,4605	0,1952
0,4662	0,3995	0,3464	0,8080	0,3883	0,6346	0,7873	0,5951
0,0516	0,5108	0,7422	0,4355	0,0495	0,0551	0,2082	0,6025
0,2702	0,9804	0,4141	0,8061	0,8924	0,3233	0,0325	0,1837
0,5394	0,6068	0,8147	0,0190	0,2475	0,5607	0,5563	0,4707
0,4049	0,9292	0,7990	0,0915	0,3177	0,3973	0,1749	0,4212
0,5596	0,1689	0,8001	0,4309	0,7600	0,2251	0,6222	0,1039
0,9888	0,7962	0,1304	0,9322	0,3926	0,0776	0,3328	0,6089
0,2394	0,7212	0,4070	0,7013	0,0272	0,1334	0,9314	0,5820
0,8144	0,1107	0,1025	0,4021	0,4256	0,8451	0,8451	0,3859
0,6925	0,8291	0,2210	0,2297	0,3822	0,6931	0,0426	0,8848
0,9728	0,5052	0,0986	0,5073	0,9304	0,5651	0,8967	0,8545
0,6440	0,2236	0,3296	0,9098	0,6753	0,7590	0,1729	0,0178
0,4791	0,8408	0,8168	0,3103	0,2504	0,6129	0,1428	0,7045
0,0551	0,9271	0,9475	0,2542	0,8997	0,7582	0,8223	0,1367
0,8972	0,3323	0,8943	0,0553	0,1846	0,9987	0,3347	0,3388
0,7486	0,3778	0,0410	0,5198	0,8161	0,8148	0,3993	0,6728
0,9648	0,6726	0,5330	0,2930	0,6117	0,4922	0,9443	0,9982
0,0334	0,5549	0,9536	0,8326	0,5776	0,8756	0,8453	0,4575
0,2184	0,1585	0,1241	0,0059	0,1155	0,7479	0,7889	0,8862
0,3299	0,1039	0,0305	0,3128	0,3550	0,8343	0,2602	0,8479
0,6184	0,8151	0,8734	0,7332	0,6102	0,4303	0,6569	0,2945
0,7947	0,4610	0,2443	0,1997	0,9619	0,6569	0,5269	0,8513
0,1740	0,5213	0,4260	0,6955	0,9646	0,3385	0,9322	0,7274

Table 8 - AREAS UNDER THE STANDARD NORMAL CURVE FROM 0 to z.



z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0753
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2257	0,2291	0,2324	0,2357	0,2389	0,2422	0,2454	0,2486	0,2517	0,2549
0,7	0,2580	0,2611	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2995	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389
1,0	0,3413	0,3438	0,3461	0,3485	0,3508	0,3531	0,3554	0,3577	0,3599	0,3621
1,1	0,3643	0,3665	0,3686	0,3708	0,3729	0,3749	0,3770	0,3790	0,3810	0,3830
1,2	0,3849	0,3869	0,3888	0,3907	0,3925	0,3944	0,3962	0,3980	0,3997	0,4015
1,3	0,4032	0,4049	0,4066	0,4082	0,4099	0,4115	0,4131	0,4147	0,4162	0,4177
1,4	0,4192	0,4207	0,4222	0,4236	0,4251	0,4265	0,4279	0,4292	0,4306	0,4319
1,5	0,4332	0,4345	0,4357	0,4370	0,4382	0,4394	0,4406	0,4418	0,4429	0,4441
1,6	0,4452	0,4463	0,4474	0,4484	0,4495	0,4505	0,4515	0,4525	0,4535	0,4545
1,7	0,4554	0,4564	0,4573	0,4582	0,4591	0,4599	0,4608	0,4616	0,4625	0,4633
1,8	0,4641	0,4649	0,4656	0,4664	0,4671	0,4678	0,4686	0,4693	0,4699	0,4706
1,9	0,4713	0,4719	0,4726	0,4732	0,4738	0,4744	0,4750	0,4756	0,4761	0,4767
2,0	0,4772	0,4778	0,4783	0,4788	0,4793	0,4798	0,4803	0,4808	0,4812	0,4817
2,1	0,4821	0,4826	0,4830	0,4834	0,4838	0,4842	0,4846	0,4850	0,4854	0,4857
2,2	0,4861	0,4864	0,4868	0,4871	0,4875	0,4878	0,4881	0,4884	0,4887	0,4890
2,3	0,4893	0,4896	0,4898	0,4901	0,4904	0,4906	0,4909	0,4911	0,4913	0,4916
2,4	0,4918	0,4920	0,4922	0,4925	0,4927	0,4929	0,4931	0,4932	0,4934	0,4936
2,5	0,4938	0,4940	0,4941	0,4943	0,4945	0,4946	0,4948	0,4949	0,4951	0,4952
2,6	0,4953	0,4955	0,4956	0,4957	0,4959	0,4960	0,4961	0,4962	0,4963	0,4964
2,7	0,4965	0,4966	0,4967	0,4968	0,4969	0,4970	0,4971	0,4972	0,4973	0,4974
2,8	0,4974	0,4975	0,4976	0,4977	0,4977	0,4978	0,4979	0,4979	0,4980	0,4981
2,9	0,4981	0,4982	0,4982	0,4983	0,4984	0,4984	0,4985	0,4985	0,4986	0,4986
3,0	0,4987	0,4987	0,4987	0,4988	0,4988	0,4989	0,4989	0,4989	0,4990	0,4990
3,1	0,4990	0,4991	0,4991	0,4991	0,4992	0,4992	0,4992	0,4992	0,4993	0,4993
3,2	0,4993	0,4993	0,4994	0,4994	0,4994	0,4994	0,4994	0,4995	0,4995	0,4995
3,3	0,4995	0,4995	0,4995	0,4996	0,4996	0,4996	0,4996	0,4996	0,4996	0,4997
3,4	0,4997	0,4997	0,4997	0,4997	0,4997	0,4997	0,4997	0,4997	0,4997	0,4998
3,5	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998
3,6	0,4998	0,4998	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999
3,7	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999
3,8	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999
3,9	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000

Table 9 - PERCENTILE VALUES (tp) FOR t-DISTRIBUTION



ν degrees of freedom

ν	α = 0,005	α = 0,001	α = 0,025	α = 0,05	α = 0,10	α = 0,20	α = 0,25	α = 0,30	α = 0,40	α = 0,45
	$t_{0,995}$	$t_{0,99}$	$t_{0,975}$	$t_{0,95}$	$t_{0,90}$	$t_{0,80}$	$t_{0,75}$	$t_{0,70}$	$t_{0,60}$	$t_{0,55}$
1	63,66	31,82	12,71	6,31	3,08	1,376	1,000	0,727	0,325	0,158
2	9,92	6,96	4,30	2,92	1,89	1,061	0,816	0,617	0,289	0,142
3	5,84	4,54	3,18	2,35	1,64	0,978	0,765	0,584	0,277	0,137
4	4,60	3,75	2,78	2,13	1,53	0,941	0,741	0,569	0,271	0,134
5	4,03	3,36	2,57	2,02	1,48	0,920	0,727	0,559	0,267	0,132
6	3,71	3,14	2,45	1,94	1,44	0,906	0,718	0,553	0,265	0,131
7	3,50	3,00	2,36	1,90	1,42	0,896	0,711	0,549	0,263	0,130
8	3,36	2,90	2,31	1,86	1,40	0,889	0,706	0,546	0,262	0,130
9	3,25	2,82	2,26	1,83	1,38	0,883	0,703	0,543	0,261	0,129
10	3,17	2,76	2,23	1,81	1,37	0,879	0,700	0,542	0,260	0,129
11	3,11	2,72	2,20	1,80	1,36	0,876	0,697	0,540	0,260	0,129
12	3,06	2,68	2,18	1,78	1,36	0,873	0,695	0,539	0,259	0,128
13	3,01	2,65	2,16	1,77	1,35	0,870	0,694	0,538	0,259	0,128
14	2,98	2,62	2,14	1,76	1,34	0,868	0,692	0,537	0,258	0,128
15	2,95	2,60	2,13	1,75	1,34	0,866	0,691	0,536	0,258	0,128
16	2,92	2,58	2,12	1,75	1,34	0,865	0,690	0,535	0,258	0,128
17	2,90	2,57	2,11	1,74	1,33	0,863	0,689	0,534	0,257	0,128
18	2,88	2,55	2,10	1,73	1,33	0,862	0,688	0,534	0,257	0,127
19	2,86	2,54	2,09	1,73	1,33	0,861	0,688	0,533	0,257	0,127
20	2,84	2,53	2,09	1,72	1,32	0,860	0,687	0,533	0,257	0,127
21	2,83	2,52	2,08	1,72	1,32	0,859	0,686	0,532	0,257	0,127
22	2,82	2,51	2,07	1,72	1,32	0,858	0,686	0,532	0,256	0,127
23	2,81	2,50	2,07	1,71	1,32	0,858	0,685	0,532	0,256	0,127
24	2,80	2,49	2,06	1,71	1,32	0,857	0,685	0,531	0,256	0,127
25	2,79	2,48	2,06	1,71	1,32	0,856	0,684	0,531	0,256	0,127
26	2,78	2,48	2,06	1,71	1,32	0,856	0,684	0,531	0,256	0,127
27	2,77	2,47	2,05	1,70	1,31	0,855	0,684	0,531	0,256	0,127
28	2,76	2,47	2,05	1,70	1,31	0,855	0,683	0,530	0,256	0,127
29	2,76	2,46	2,04	1,70	1,31	0,854	0,683	0,530	0,256	0,127
30	2,75	2,46	2,04	1,70	1,31	0,854	0,683	0,530	0,256	0,127
40	2,70	2,42	2,02	1,68	1,30	0,851	0,681	0,529	0,255	0,126
60	2,66	2,39	2,00	1,67	1,30	0,848	0,679	0,527	0,254	0,126
120	2,62	2,36	1,98	1,66	1,29	0,845	0,677	0,526	0,254	0,126

Table 10 (a) - PERCENTAGE VALUES OF THE F-DISTRIBUTION

$$f_{0,05 \;\; v1 \;\; v2}$$

	v_1																		
v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	□
1	161	199	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18,51	19,00	19,16	19,25	19,30	19,33	19,35	19,37	19,38	19,40	19,41	19,43	19,45	19,45	19,46	19,47	19,48	19,49	19,50
3	10,13	9,55	9,28	9,12	9,01	8,94	8,89	8,85	8,81	8,79	8,74	8,70	8,66	8,64	8,62	8,59	8,57	8,55	8,53
4	7,71	6,94	6,59	6,39	6,26	6,16	6,09	6,04	6,00	5,96	5,91	5,86	5,80	5,77	5,75	5,72	5,69	5,66	5,63
5	6,61	5,79	5,41	5,19	5,05	4,95	4,88	4,82	4,77	4,74	4,68	4,62	4,56	4,53	4,50	4,46	4,43	4,40	4,37
6	5,99	5,14	4,76	4,53	4,39	4,28	4,21	4,15	4,10	4,06	4,00	3,94	3,87	3,84	3,81	3,77	3,74	3,70	3,67
7	5,59	4,74	4,35	4,12	3,97	3,87	3,79	3,73	3,68	3,64	3,57	3,51	3,44	3,41	3,38	3,34	3,30	3,27	3,23
8	5,32	4,46	4,07	3,84	3,69	3,58	3,50	3,44	3,39	3,35	3,28	3,22	3,15	3,12	3,08	3,04	3,01	2,97	2,93
9	5,12	4,26	3,86	3,63	3,48	3,37	3,29	3,23	3,18	3,14	3,07	3,01	2,94	2,90	2,86	2,83	2,79	2,75	2,71
10	4,96	4,10	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,91	2,85	2,77	2,74	2,70	2,66	2,62	2,58	2,54
11	4,84	3,98	3,59	3,36	3,20	3,09	3,01	2,95	2,90	2,85	2,79	2,72	2,65	2,61	2,57	2,53	2,49	2,45	2,40
12	4,75	3,89	3,49	3,26	3,11	3,00	2,91	2,85	2,80	2,75	2,69	2,62	2,54	2,51	2,47	2,43	2,38	2,34	2,30
13	4,67	3,81	3,41	3,18	3,03	2,92	2,83	2,77	2,71	2,67	2,60	2,53	2,46	2,42	2,38	2,34	2,30	2,25	2,21
14	4,60	3,74	3,34	3,11	2,96	2,85	2,76	2,70	2,65	2,60	2,53	2,46	2,39	2,35	2,31	2,27	2,22	2,18	2,13
15	4,54	3,68	3,29	3,06	2,90	2,79	2,71	2,64	2,59	2,54	2,48	2,40	2,33	2,29	2,25	2,20	2,16	2,11	2,07
16	4,49	3,63	3,24	3,01	2,85	2,74	2,66	2,59	2,54	2,49	2,42	2,35	2,28	2,24	2,19	2,15	2,11	2,06	2,01
17	4,45	3,59	3,20	2,96	2,81	2,70	2,61	2,55	2,49	2,45	2,38	2,31	2,23	2,19	2,15	2,10	2,06	2,01	1,96
18	4,41	3,55	3,16	2,93	2,77	2,66	2,58	2,51	2,46	2,41	2,34	2,27	2,19	2,15	2,11	2,06	2,02	1,97	1,93
19	4,38	3,52	3,13	2,90	2,74	2,63	2,54	2,48	2,42	2,38	2,31	2,23	2,16	2,11	2,07	2,03	1,98	1,93	1,88
20	4,35	3,49	3,10	2,87	2,71	2,60	2,51	2,45	2,39	2,35	2,28	2,20	2,12	2,08	2,04	1,99	1,95	1,90	1,84
21	4,32	3,47	3,07	2,84	2,68	2,57	2,49	2,42	2,37	2,32	2,25	2,18	2,10	2,05	2,01	1,96	1,92	1,87	1,81
22	4,30	3,44	3,05	2,82	2,66	2,55	2,46	2,40	2,34	2,30	2,23	2,15	2,07	2,03	1,98	1,94	1,89	1,84	1,78
23	4,28	3,42	3,03	2,80	2,64	2,53	2,44	2,37	2,32	2,27	2,20	2,13	2,05	2,01	1,96	1,91	1,86	1,81	1,76
24	4,26	3,40	3,01	2,78	2,62	2,51	2,42	2,36	2,30	2,25	2,18	2,11	2,03	1,98	1,94	1,89	1,84	1,79	1,73
25	4,24	3,39	2,99	2,76	2,60	2,49	2,40	2,34	2,28	2,24	2,16	2,09	2,01	1,96	1,92	1,87	1,82	1,77	1,71
30	4,17	3,32	2,92	2,69	2,53	2,42	2,33	2,27	2,21	2,16	2,09	2,01	1,93	1,89	1,84	1,79	1,74	1,68	1,62
40	4,08	3,23	2,84	2,61	2,45	2,34	2,25	2,18	2,12	2,08	2,00	1,92	1,84	1,79	1,74	1,69	1,64	1,58	1,51
60	4,00	3,15	2,76	2,53	2,37	2,25	2,17	2,10	2,04	1,99	1,92	1,84	1,75	1,70	1,65	1,59	1,53	1,47	1,39
120	3,92	3,07	2,68	2,45	2,29	2,18	2,09	2,02	1,96	1,91	1,83	1,75	1,66	1,61	1,55	1,50	1,43	1,35	1,25
∞	3,84	3,00	2,60	2,37	2,21	2,10	2,01	1,94	1,88	1,83	1,75	1,67	1,57	1,52	1,46	1,39	1,32	1,22	1,00

Table 10 (b) - PERCENTAGE VALUES OF THE F-DISTRIBUTION

$f_{0,01\ v1\ v2}$

	v_1																		
v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98,50	99,00	99,17	99,25	99,30	99,33	99,36	99,37	99,39	99,40	99,42	99,43	99,45	99,46	99,47	99,47	99,48	99,49	99,50
3	34,12	30,82	29,46	28,71	28,24	27,91	27,67	27,49	27,35	27,23	27,05	26,87	26,69	26,60	26,50	26,41	26,32	26,22	26,10
4	21,20	18,00	16,69	15,98	15,52	15,21	14,98	14,80	14,66	14,55	14,37	14,20	14,02	13,93	13,84	13,75	13,65	13,56	13,50
5	16,26	13,27	12,06	11,39	10,97	10,67	10,46	10,29	10,16	10,05	9,89	9,72	9,55	9,47	9,38	9,29	9,20	9,11	9,02
6	13,75	10,92	9,78	9,15	8,75	8,47	8,26	8,10	7,98	7,87	7,72	7,56	7,40	7,31	7,23	7,14	7,06	6,97	6,88
7	12,25	9,55	8,45	7,85	7,46	7,19	6,99	6,84	6,72	6,62	6,47	6,31	6,16	6,07	5,99	5,91	5,82	5,74	5,65
8	11,26	8,65	7,59	7,01	6,63	6,37	6,18	6,03	5,91	5,81	5,67	5,52	5,36	5,28	5,20	5,12	5,03	4,95	4,83
9	10,56	8,02	6,99	6,42	6,06	5,80	5,61	5,47	5,35	5,26	5,11	4,96	4,81	4,73	4,65	4,57	4,48	4,40	4,31
10	10,04	7,56	6,55	5,99	5,64	5,39	5,20	5,06	4,94	4,85	4,71	4,56	4,41	4,33	4,25	4,17	4,08	4,00	3,91
11	9,65	7,21	6,22	5,67	5,32	5,07	4,89	4,74	4,63	4,54	4,40	4,25	4,10	4,02	3,94	3,86	3,78	3,69	3,60
12	9,33	6,93	5,95	5,41	5,06	4,82	4,64	4,50	4,39	4,30	4,16	4,01	3,86	3,78	3,70	3,62	3,54	3,45	3,36
13	9,07	6,70	5,74	5,21	4,86	4,62	4,44	4,30	4,19	4,10	3,96	3,82	3,66	3,59	3,51	3,43	3,34	3,25	3,17
14	8,86	6,51	5,56	5,04	4,69	4,46	4,28	4,14	4,03	3,94	3,80	3,66	3,51	3,43	3,35	3,27	3,18	3,09	3,00
15	8,68	6,36	5,42	4,89	4,56	4,32	4,14	4,00	3,89	3,80	3,67	3,52	3,37	3,29	3,21	3,13	3,05	2,96	2,87
16	8,53	6,23	5,29	4,77	4,44	4,20	4,03	3,89	3,78	3,69	3,55	3,41	3,26	3,18	3,10	3,02	2,93	2,84	2,75
17	8,40	6,11	5,18	4,67	4,34	4,10	3,93	3,79	3,68	3,59	3,46	3,31	3,16	3,08	3,00	2,92	2,83	2,75	2,65
18	8,29	6,01	5,09	4,58	4,25	4,01	3,84	3,71	3,60	3,51	3,37	3,23	3,08	3,00	2,92	2,84	2,75	2,66	2,57
19	8,18	5,93	5,01	4,50	4,17	3,94	3,77	3,63	3,52	3,43	3,30	3,15	3,00	2,92	2,84	2,76	2,67	2,58	2,49
20	8,10	5,85	4,94	4,43	4,10	3,87	3,70	3,56	3,46	3,37	3,23	3,09	2,94	2,86	2,78	2,69	2,61	2,52	2,42
21	8,02	5,78	4,87	4,37	4,04	3,81	3,64	3,51	3,40	3,31	3,17	3,03	2,88	2,80	2,72	2,64	2,55	2,46	2,36
22	7,95	5,72	4,82	4,31	3,99	3,76	3,59	3,45	3,35	3,26	3,12	2,98	2,83	2,75	2,67	2,58	2,50	2,40	2,31
23	7,88	5,66	4,76	4,26	3,94	3,71	3,54	3,41	3,30	3,21	3,07	2,93	2,78	2,70	2,62	2,54	2,45	2,35	2,26
24	7,82	5,61	4,72	4,22	3,90	3,67	3,50	3,36	3,26	3,17	3,03	2,89	2,74	2,66	2,58	2,49	2,40	2,31	2,21
25	7,77	5,57	4,68	4,18	3,85	3,63	3,46	3,32	3,22	3,13	2,99	2,85	2,70	2,62	2,54	2,45	2,36	2,27	2,17
30	7,56	5,39	4,51	4,02	3,70	3,47	3,30	3,17	3,07	2,98	2,84	2,70	2,55	2,47	2,39	2,30	2,21	2,11	2,01
40	7,31	5,18	4,31	3,83	3,51	3,29	3,12	2,99	2,89	2,80	2,66	2,52	2,37	2,29	2,20	2,11	2,02	1,92	1,80
60	7,08	4,98	4,13	3,65	3,34	3,12	2,95	2,82	2,72	2,63	2,50	2,35	2,20	2,12	2,03	1,94	1,84	1,73	1,60
120	6,85	4,79	3,95	3,48	3,17	2,96	2,79	2,66	2,56	2,47	2,34	2,19	2,03	1,95	1,86	1,76	1,66	1,53	1,38
∞	6,63	4,61	3,78	3,32	3,02	2,80	2,64	2,51	2,41	2,32	2,18	2,04	1,88	1,79	1,70	1,59	1,47	1,32	1,00

Table 11 (a) - ERROR AND UNCERTAINTY

Material or Product	Category	Property	Unit	Maximum permissible error E	Uncertainty (σ)
Asphalt	Densification	Density (Marshall)	kg/m^3	5,0	7
		Compaction (Marshall)	%	2,0	2,5
		Density (Rice)	kg/m^3	3,5	5
		Compaction (Rice)	%	2,0	2
		Density - core	kg/m^3		7
	Gradation (percent passing)	13,2 mm	%	2,5	3
		9,5 mm	%	2,5	3
		4,75 mm	%	1,8	2
		2,36 mm	%	1,5	2
		0,30 mm	%	1,0	2
		0,075 mm	%	0,5	1
	Miscellaneous	Binder content	%	0,2	0,3
		Voids in mix	%	0,8	1
Concrete	Cube strength (Mpa)[3]	Small structures (small culverts)	MPa	5	7*
		Medium size structures (small bridges)	MPa	4,5	6*
		Large structures	MPa	3	5*
	Coarse aggregate	37,5 mm	%	2,5	3
		26,5 mm	%	3	5
		19,0 mm	%	3	5
		13,2 mm	%	2,5	4
		9,5 mm	%	2,5	3
		6,7 mm	%	2,5	3
	Fine aggregate	4,75 mm	%	2	3
		2,36 mm	%	2	3
		1,18 mm	%	2	3
		0,600 mm	%	1,8	2
		0,425 mm	%	1,8	2
		0,300 mm	%	1,8	2
		0,150 mm	%	1,2	1,5
		0,075 mm	%	1	1

[3] Fulton's Concrete Technology, 8th edition, 2001

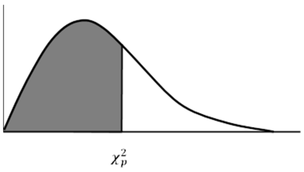
Table (b) - ERROR AND UNCERTAINTY

Material or Product	Category	Property	Unit	Maximum permissible error E	Uncertainty (σ)
Layer (G1, G2 & G3)	Densification	Density – Nuclear	kg/m^3	35	45
		Compaction	%	1,5	2
	Gradation (percent passing)	26,5 mm	%	1,5	2
		19,5 mm	%	2,5	5
		13,2 mm	%	2	3
		4,75 mm	%	2	3
		2,0 mm	%	1,5	2
		0,425 mm	%	1,5	2
		0,075 mm	%	1	1
	Atterberg Limits	L.L.	%	1	1,5
		P.L.	%	1	1,5
		P.I.	%	2	2
		L.S.	%	0,5	1
Layer (G4, G5 & G6)	Densification	Density – Nuclear	kg/m^3	37	48
		Compaction	%	1,7	2,3
	Gradation (percent passing)	26,5 mm	%	3	5
		19,5 mm	%	2,5	5
		13,2 mm	%	3,5	5
		4,75 mm	%	2,5	4
		2,0 mm	%	2	3
		0,425 mm	%	1,8	2
		0,075 mm	%	1,2	2
	Atterberg limits	L.L.	%	1,5	2
		P.L.	%	2	2,5
		P.I.	%	2,5	3
		L.S.	%	1,5	2
Layer (G7, G8 & G9)	Densification	Density – Nuclear		40	50
		Compaction		2	2,5
	Gradation (percent passing)	26,5 mm		3,5	5
		19,5 mm		3	5
		13,2 mm		3,5	5
		4,75 mm		3	4
		2,0 mm		2,5	3
		0,425 mm		2	2,5
		0,075 mm		1,8	2
	Atterberg limits	L.L.		1,8	2,5
		P.L.		2	2,5
		P.I.	%	2,5	3
		L.S.	%	1,5	2

Table - ERROR AND UNCERTAINTY

Material or Product	Category	Property	Unit	Maximum permissible error E	Uncertainty (σ)
Layer (G10)	Densification	Density – Nuclear	kg/m^3	35	55
		Compaction	%	2	2,5
	Gradation (percent passing)	26,5 mm	%	3	5
		19,5 mm	%	3	5
		13,2 mm	%	3,5	5
		4,75 mm	%	2,5	4
		2,0 mm	%	2	4
		0,425 mm	%	2	3
		0,075 mm	%	1,5	2,5
	Atterberg limits	L.L.	%	1,5	2,7
		P.L.	%	2	2,7
		P.I.	%	2,5	3,5
		L.S.	%	1,5	2
Surfacing aggregate	Gradation (percent passing)	26,5 mm	%	2,5	2,5
		19,5 mm	%	2,5	2,5
		13,2 mm	%	2,5	2,5
		9,5 mm	%	2,5	2,5
		6,7 mm	%	2	2
		4,75 mm	%	2	2
		3,35 mm	%	1,5	1,5
		2,36 mm	%	1,5	1,5
		1,18 mm	%	1,5	1,5
		0,600 mm	%	0,8	1
		0,300 mm	%	0,8	1
		0,150 mm	%	0,8	1
		0,075 mm	%	0,5	0,5
	Miscellaneous	Flakiness index	%	1,5	2
		A.L.D. measured Same test portion	mm	0,5	0,5
		A.L.D. measured Different test portion	mm	0,7	1

Table 12 - PERCENTILE VALUES (χ_P^2) FOR THE CHI-SQUARE DISTRIBUTION



With ν Degrees of Freedom

Shaded area = p

ν	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{..01}$	$\chi^2_{.005}$
1	7.88	6.64	5.02	3.84	2.71	0.0158	0.0040	0.0010	0.0002	0.0000
2	10.6	9.21	7.38	5.99	4.61	0.211	0.103	0.0506	0.0201	0.0100
3	12.8	11.3	9.35	7.82	6.25	0.584	0.352	0.216	0.115	0.072
4	14.9	13.3	11.1	9.49	7.78	1.06	0.711	0.484	0.297	0.207
5	16.8	15.1	12.8	11.1	9.24	1.61	1.15	0.831	0.554	0.412
6	18.5	16.8	14.4	12.6	10.6	2.20	1.64	1.24	0.872	0.676
7	20.3	18.5	16.0	14.1	12.0	2.83	2.17	1.69	1.24	0.989
8	22.0	20.1	17.5	15.5	13.4	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	4.17	3.33	2.70	2.09	1.74
10	25.2	23.2	20.5	18.3	16.0	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	5.58	4.58	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	7.79	6.57	5.63	4.66	4.08
15	32.8	30.6	27.5	1.0	22.3	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	10.9	9.39	8.23	7.02	6.27
19	38.6	36.2	32.9	30.1	27.2	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	13.2	11.6	10.3	8.90	8.03
22	42.8	40.3	36.8	33.9	30.8	14.0	12.3	11.0	9.54	8.64
23	44.2	41.6	38.1	35.2	32.0	14.8	13.1	11.7	10.2	9.26
24	45.6	43.0	39.4	36.4	33.2	15.7	13.8	12.4	10.9	9.89
25	46.9	44.3	40.6	37.7	34.4	16.5	14.6	13.1	11.5	10.5
26	48.3	45.6	41.9	38.9	35.6	17.3	15.4	13.8	12.2	11.2
27	49.6	47.0	43.2	40.1	36.7	18.1	16.2	14.6	12.9	11.8
28	51.0	48.3	44.5	41.3	37.9	18.9	16.9	15.3	13.6	12.5
29	52.3	49.6	45.7	42.6	39.1	19.8	17.7	16.0	14.3	13.1
30	53.7	50.9	47.0	43.8	40.3	20.6	18.5	16.8	15.0	13.8
40	66.8	63.7	59.3	55.8	51.8	29.1	26.5	24.4	22.2	20.7
50	79.5	76.2	71.4	67.5	63.2	37.7	34.8	32.4	29.7	28.0
60	92.0	88.4	83.3	79.1	74.4	46.5	43.2	40.5	37.5	35.5
70	104.2	100.4	95.0	90.5	85.5	55.3	51.7	48.8	45.4	43.3
80	116.3	112.3	106.6	101.9	96.6	64.3	60.4	57.2	53.5	51.2
90	128.3	124.1	118.1	113.1	107.6	73.3	69.1	65.6	61.8	59.2
100	140.2	135.8	129.6	124.3	118.5	82.4	77.9	74.2	70.1	67.3

Table 13 - CRITICAL VALUES FOR $C_{0,01;\,n}$

n	C	n	C
3	2,22	7	2,76
4	2,43	8	2,83
5	2,57	9	2,88
6	2,68	>9	2,90

Table 14 (a) - F_A-VALUES FOR SINGLE LIMIT SPECIFICATION.

(α = 5%)

n	φ-value							
	10	12	14	15	16	18	20	25
3	0,337	0,237	0,143	0,096	0,051	-0,040	-0,132	-0,374
4	0,445	0,350	0,262	0,220	0,179	0,099	0,021	-0,171
5	0,520	0,426	0,341	0,300	0,260	0,184	0,110	-0,066
6	0,576	0,483	0,398	0,358	0,319	0,244	0,173	0,003
7	0,620	0,527	0,443	0,403	0,365	0,291	0,221	0,055
8	0,656	0,563	0,479	0,440	0,401	0,328	0,258	0,095
9	0,687	0,594	0,510	0,470	0,432	0,359	0,290	0,128
10	0,713	0,620	0,535	0,496	0,458	0,385	0,316	0,155
11	0,735	0,642	0,558	0,518	0,480	0,408	0,339	0,179
12	0,755	0,662	0,577	0,538	0,500	0,427	0,359	0,199
13	0,773	0,679	0,595	0,555	0,517	0,445	0,376	0,217
14	0,789	0,695	0,610	0,571	0,533	0,460	0,392	0,233
15	0,803	0,709	0,625	0,585	0,547	0,474	0,406	0,247
16	0,817	0,722	0,637	0,598	0,560	0,487	0,419	0,260
17	0,829	0,734	0,649	0,610	0,571	0,499	0,430	0,272
18	0,840	0,745	0,660	0,620	0,582	0,509	0,441	0,283
19	0,850	0,755	0,670	0,630	0,592	0,519	0,451	0,293
20	0,859	0,765	0,679	0,639	0,601	0,528	0,460	0,302
21	0,868	0,773	0,688	0,648	0,610	0,537	0,468	0,310
22	0,877	0,781	0,696	0,656	0,618	0,545	0,476	0,318
23	0,885	0,789	0,704	0,664	0,625	0,552	0,484	0,326
24	0,892	0,796	0,711	0,671	0,632	0,559	0,491	0,333
25	0,899	0,803	0,717	0,677	0,639	0,566	0,497	0,339
26	0,905	0,810	0,724	0,684	0,645	0,572	0,503	0,346
27	0,912	0,816	0,730	0,690	0,651	0,578	0,509	0,351
28	0,918	0,821	0,735	0,695	0,657	0,584	0,515	0,357
29	0,923	0,827	0,741	0,701	0,662	0,589	0,520	0,362
30	0,928	0,832	0,746	0,706	0,667	0,594	0,525	0,367

Table 14 (b) - F_R VALUES FOR SINGLE LIMIT SPECIFICATION.

(α = 1%)

n	φ-value							
	10	12	14	15	16	18	20	25
4	0,123	0,013	-0,095	-0,148	-0,202	-0,312	-0,428	-0,762
5	0,238	0,137	0,042	-0,004	-0,050	-0,140	-0,231	-0,463
6	0,320	0,223	0,132	0,089	0,047	-0,037	-0,118	-0,321
7	0,382	0,287	0,199	0,158	0,117	0,037	-0,040	-0,228
8	0,432	0,338	0,252	0,211	0,171	0,094	0,020	-0,160
9	0,474	0,381	0,295	0,255	0,216	0,140	0,067	-0,107
10	0,509	0,416	0,332	0,292	0,253	0,178	0,107	-0,064
11	0,540	0,447	0,363	0,323	0,285	0,211	0,140	-0,028
12	0,567	0,474	0,390	0,351	0,312	0,239	0,169	0,003
13	0,590	0,498	0,414	0,375	0,337	0,264	0,194	0,030
14	0,612	0,520	0,436	0,397	0,359	0,286	0,216	0,053
15	0,631	0,539	0,455	0,416	0,378	0,305	0,236	0,074
16	0,649	0,557	0,473	0,434	0,396	0,323	0,254	0,093
17	0,665	0,573	0,489	0,450	0,412	0,339	0,271	0,110
18	0,680	0,587	0,504	0,465	0,427	0,354	0,286	0,126
19	0,694	0,601	0,517	0,478	0,440	0,368	0,300	0,140
20	0,706	0,614	0,530	0,491	0,453	0,381	0,312	0,153
21	0,718	0,626	0,542	0,503	0,465	0,393	0,324	0,165
22	0,729	0,637	0,553	0,513	0,476	0,404	0,335	0,177
23	0,740	0,647	0,563	0,524	0,486	0,414	0,346	0,187
24	0,750	0,657	0,573	0,533	0,496	0,423	0,355	0,197
25	0,759	0,666	0,582	0,542	0,505	0,432	0,364	0,206
26	0,768	0,674	0,590	0,551	0,513	0,441	0,373	0,215
27	0,776	0,683	0,599	0,559	0,521	0,449	0,381	0,223
28	0,784	0,690	0,606	0,567	0,529	0,457	0,389	0,231
29	0,791	0,698	0,614	0,574	0,536	0,464	0,396	0,238
30	0,799	0,705	0,621	0,581	0,543	0,471	0,403	0,245

Table 14 (c) - F_A-VALUES FOR DOUBLE LIMIT SPECIFICATION

(α = 5%)

n	φ-value							
	10	12	14	15	16	18	20	25
4	0,299	0,199	0,106	0,061	0,016	-0,073	-0,161	-0,387
5	0,390	0,294	0,206	0,163	0,122	0,041	-0,038	-0,232
6	0,456	0,362	0,276	0,235	0,195	0,118	0,043	-0,137
7	0,508	0,415	0,330	0,290	0,251	0,175	0,103	-0,070
8	0,551	0,458	0,374	0,334	0,295	0,221	0,150	-0,019
9	0,586	0,494	0,410	0,370	0,332	0,258	0,188	0,022
10	0,617	0,524	0,440	0,401	0,363	0,289	0,220	0,056
11	0,643	0,551	0,467	0,427	0,389	0,316	0,247	0,085
12	0,666	0,574	0,490	0,451	0,412	0,340	0,271	0,109
13	0,687	0,594	0,510	0,471	0,433	0,361	0,292	0,131
14	0,705	0,613	0,529	0,489	0,451	0,379	0,310	0,150
15	0,722	0,629	0,545	0,506	0,468	0,396	0,327	0,168
16	0,737	0,644	0,560	0,521	0,483	0,411	0,342	0,183
17	0,751	0,658	0,574	0,535	0,497	0,424	0,356	0,197
18	0,764	0,671	0,587	0,547	0,509	0,437	0,369	0,210
19	0,776	0,683	0,598	0,559	0,521	0,449	0,380	0,222
20	0,787	0,694	0,609	0,570	0,532	0,459	0,391	0,233
21	0,798	0,704	0,619	0,580	0,542	0,470	0,401	0,243
22	0,807	0,713	0,629	0,589	0,551	0,479	0,411	0,253
23	0,816	0,722	0,638	0,598	0,560	0,488	0,419	0,262
24	0,825	0,731	0,646	0,606	0,568	0,496	0,428	0,270
25	0,833	0,739	0,654	0,614	0,576	0,504	0,435	0,278
26	0,841	0,746	0,661	0,621	0,583	0,511	0,443	0,285
27	0,848	0,753	0,668	0,628	0,590	0,518	0,449	0,292
28	0,855	0,760	0,675	0,635	0,597	0,524	0,456	0,298
29	0,861	0,766	0,681	0,641	0,603	0,531	0,462	0,305
30	0,867	0,772	0,687	0,647	0,609	0,536	0,468	0,311

Table 14 (d) - F_R VALUES FOR DOUBLE LIMIT SPECIFICATION

(α = 1%)

n	φ-value							
	10	12	14	15	16	18	20	25
4	-0,007	-0,132	-0,258	-0,324	-0,392	-0,538	-0,707	-1,430
5	0,132	0,024	-0,079	-0,130	-0,181	-0,283	-0,389	-0,675
6	0,226	0,125	0,030	-0,016	-0,061	-0,151	-0,240	-0,468
7	0,296	0,199	0,108	0,064	0,022	-0,062	-0,144	-0,348
8	0,352	0,257	0,168	0,126	0,085	0,005	-0,073	-0,264
9	0,398	0,304	0,217	0,176	0,136	0,058	-0,018	-0,200
10	0,437	0,344	0,258	0,217	0,178	0,101	0,028	-0,149
11	0,471	0,378	0,293	0,253	0,214	0,138	0,066	-0,107
12	0,500	0,408	0,323	0,283	0,244	0,170	0,098	-0,072
13	0,526	0,434	0,350	0,310	0,272	0,198	0,127	-0,041
14	0,550	0,458	0,374	0,334	0,296	0,222	0,152	-0,014
15	0,571	0,479	0,395	0,356	0,317	0,244	0,174	0,010
16	0,590	0,498	0,414	0,375	0,337	0,264	0,194	0,031
17	0,608	0,516	0,432	0,393	0,355	0,282	0,213	0,050
18	0,624	0,532	0,448	0,409	0,371	0,299	0,229	0,068
19	0,639	0,547	0,463	0,424	0,386	0,314	0,245	0,084
20	0,653	0,561	0,477	0,438	0,400	0,328	0,259	0,098
21	0,666	0,574	0,490	0,451	0,413	0,341	0,272	0,112
22	0,678	0,586	0,502	0,463	0,425	0,353	0,284	0,125
23	0,689	0,597	0,513	0,474	0,436	0,364	0,296	0,136
24	0,700	0,607	0,524	0,485	0,447	0,375	0,306	0,147
25	0,710	0,617	0,534	0,495	0,457	0,385	0,317	0,158
26	0,720	0,627	0,543	0,504	0,466	0,394	0,326	0,167
27	0,729	0,636	0,552	0,513	0,475	0,403	0,335	0,176
28	0,737	0,644	0,561	0,521	0,484	0,411	0,343	0,185
29	0,745	0,652	0,569	0,529	0,492	0,419	0,351	0,193
30	0,753	0,660	0,576	0,537	0,499	0,427	0,359	0,201

Table 15 - CONTROL CHART CONSTANTS

Sample size	$\overline{X}$ - Chart			R - Chart				
	Factors for control limits			Factors for central line	Factors for control limits			
n	A	A_1	A_2	d_2	D_1	D_2	D_3	D_4
2	2,121	3,760	1,880	1,128	0	3,686	0	3,267
3	1,732	2,394	1,023	1,693	0	4,358	0	2,575
4	1,500	1,880	0,729	2,059	0	4,698	0	2,282
5	1,342	1,596	0,577	2,326	0	4,918	0	2,115
6	1,225	1,410	0,483	2,534	0	5,078	0	2,004
7	1,134	1,277	0,419	2,704	0,205	5,203	0,076	1,924
8	1,061	1,175	0,373	2,847	0,387	5,307	0,136	1,864
9	1,000	1,094	0,337	2,970	0,546	5,394	0,184	1,816
10	0,949	1,028	0,308	3,078	0,687	5,469	0,223	1,777
11	0,905	0,973	0,285	3,173	0,812	5,534	0,256	1,744
12	0,866	0,925	0,266	3,258	0,924	5,592	0,284	1,716
13	0,832	0,884	0,249	3,336	1,026	5,646	0,308	1,692
14	0,802	0,848	0,235	3,407	1,121	5,693	0,329	1,671
15	0,775	0,816	0,223	3,472	1,207	5,737	0,348	1,652
16		0,788	0,212	3,532			0,364	1,636
17		0,762	0,203	3,588			0,379	1,621
18		0,738	0,194	3,640			0,392	1,608
19		0,717	0,187	3,689			0,404	1,596
20		0,697	0,180	3,735			0,414	1,586
21		0,679	0,173	3,778			0,425	1,575
22		0,662	0,167	3,819			0,434	1,566
23		0,647	0,162	3,858			0,443	1,557
24		0,632	0,157	3,895			0,452	1,548
25		0,619	0,153	3,931			0,459	1,541